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Physics - Understanding models, solving problems and discussing results

An in-depth guide, available offline

Read, understand, return to a chapter

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Physics - Understanding models, solving problems and discussing results

A substantial guide for active study

This guide is designed to deepen understanding of physics at the level of the French general-track final-year specialist course, revisiting the necessary earlier tools. It does not cover all the chemistry in the physics and chemistry curriculum and is not an official list of examination questions. Institutional references provide a framework; the explanations, fictional situations, demonstrations and exercises are original LFDP writing. Codex, OpenAI's artificial intelligence assistant, contributed to the drafting and English translation. The French manuscript has been independently reviewed and corrected. The few extensions beyond the central expectations are identified as such in the text.

The aim is to move from a situation to a model, from the model to relationships, and from those relationships to a defensible conclusion. A correct numerical value without clear assumptions may answer the wrong question. An explanation without calculation may remain too vague when the data allow an estimate. The chapters therefore connect ideas, arguments, illustrations and checks, so that formulas are neither isolated recipes nor an obstacle to understanding.

The examples of bodies, sources and devices are educational. They do not describe a product's guaranteed performance, prescribe construction or replace a supervised experimental procedure. Rounded values of g, fictional materials and simplifications are identified. For a real experiment, the institution's instructions, supervision and the device's characteristics remain essential.

Organisation and reading routes

The first chapter establishes quantities, units and measurement. Chapters two to five develop motion, interactions, energy and gravitation. The sixth addresses fluids; the seventh electrical circuits and RC behaviour; the eighth thermodynamics. Chapters nine to eleven connect waves, optics and photons. The final chapter brings together methods of solving and discussing problems. The appendices offer additional problems, with explained answers and distinctions that span several themes.

For a first reading, follow this order, because reasoning about motion and energy reappears later. For targeted revision, begin with a question you cannot answer, then return to the definition or model on which the answer depends. Each chapter is organised into short sections so that explanations can be found quickly on a phone or in a printed version. This document structure does not guarantee the functioning of a particular software reader.

A study session can have three stages: read a section without trying to memorise everything; close the text and explain the idea in a few sentences; solve an exercise, stating the assumptions before comparing your work with the answer. If your answer differs, first find where the reasoning diverges. Changing only the final result does not turn an error into learning.

Conventions and using worked answers

Equations use symbols defined in their context. The sign of a force, a velocity or an energy exchange depends on the stated convention. A letter may represent different quantities in two chapters; the local definition takes priority. Powers of ten and prefixes must be converted before numerical substitution. The centred dot in a unit, such as J·s, represents multiplication; a negative exponent represents a reciprocal.

A worked answer is an example of an approach, not the only acceptable account. Other routes can be equivalent if they use consistent assumptions and answer the same question. However, two similar values are insufficient to establish the equivalence of two models. Checks of dimensions, signs, orders of magnitude and initial conditions must accompany the calculation.

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Chapter 1 - Measuring, modelling and discussing uncertainty

Objectives: distinguish a quantity from a unit, choose a system and a model, read a measurement and explain a limitation. Later chapters develop the mechanisms introduced by these first situations.

Historical reference point - 1960: naming a common system of measurement

In 1960, the eleventh General Conference on Weights and Measures gave the system based on the base units then adopted the name International System of Units and the abbreviation SI. This institutional reference point shows that a unit is not an individual preference: it belongs to a shared organisation of measurement. The dimensional reasoning in this chapter uses that coherence; it does not depend on the country where a result is written. Adopting a common language nevertheless eliminates neither the uncertainty of a particular measurement nor the limits of the model used.

Reference for the historical fact, consulted for the French manuscript on 11 October 2026: BIPM, resolution 12 of the 11th CGPM, 1960, https://www.bipm.org/fr/committees/cg/cgpm/11-1960/resolution-12. LFDP educational analysis, without verbatim quotation or reproduced image.

1. Begin with a question that can be examined

“This object is moving fast” expresses an impression; “its position increases by two metres in one second in this frame of reference” describes a checkable result. Physics therefore begins not by choosing a formula, but by specifying the system studied, the quantity sought and the observation conditions. Studying a trolley requires deciding whether to include its load, defining a position origin and stating how time is measured.

An experiment and a model meet different needs. An experiment provides observations made in particular circumstances; a model organises relationships that explain them or predict other results. A particle model can be useful for a trolley's displacement without describing the rotation of its wheels. This simplification is not a mistake if it is stated and suited to the question. It becomes misleading when asked to describe what it deliberately left out.

To prepare a solution, write four lines: system, frame of reference, assumptions, unknown. These often prevent more errors than a long list of formulas. If the unknown is an energy, obtaining only a velocity does not yet answer the question.

2. A quantity is neither its numerical value nor its unit

A length of 1.20 m and a length of 120 cm are identical. The numerical value changes when the unit changes, whereas the quantity remains the same. The International System connects measurements; the metre, second and kilogram serve as reference units here. Distinguish mass, expressed in kilograms, from weight, a force expressed in newtons.

Dimensions provide a check independent of numerical calculation. In a relationship of the form distance = speed × duration, the product (m/s) × s does indeed give metres. An expression adding a length to a time has no meaning in this framework. An expression

can nevertheless have the correct dimensions and still be physically false: dimensional analysis eliminates errors but does not demonstrate a law.

Convert 72 km/h: 72 × 1000 / 3600 = 20 m/s. Forgetting this conversion before calculating kinetic energy amplifies the error, because the speed will be squared. Keep units in intermediate stages: they show where reasoning has changed its nature.

3. Measurement, scatter and uncertainty

A display of 1.20 m does not mean that the length is known exactly. Instrument resolution, the choice of start and end points, calibration and reading may all matter. Repeating a measurement provides information about its scatter; it does not automatically reveal a bias common to all measurements. An instrument with an offset can produce results very close to one another but all displaced from the correct value.

Suppose two results are 1.20 m and 1.21 m. Without information about their uncertainties, we cannot conclude that the objects have different lengths. If the exercise gives comparison margins of 0.02 m, their ranges of plausible values overlap. This observation calls for caution; it does not prove exact equality of the quantities either. An illustrative margin must not be turned into a statistical confidence interval without a method justifying that interpretation.

The precision of reported digits should follow that of the data. Writing an acceleration to ten decimal places because the calculator displays them does not produce ten decimal places of information. Keep enough digits during the calculation, then round the final answer, explaining the precision retained.

4. Describe motion before seeking its cause

Position depends on the frame of reference. A person sitting in a train is stationary relative to the seat and moving relative to the platform. Choosing an Earth-fixed frame for an exercise near the ground is often convenient; treating it as inertial remains an approximation, to be discussed according to the scale and required precision.

For straight-line motion, average velocity is displacement divided by duration: v_moy = Δx / Δt. Displacement has a direction; it is not the total distance travelled. A return journey can give zero displacement despite a non-zero distance. Instantaneous velocity is the derivative of position: v(t) = x'(t). Acceleration describes the change in velocity: a(t) = v'(t).

Consider x(t) = 0.40 t² + 0.20 t, with x in metres and t in seconds. The coefficients carry the necessary units. We obtain v(t) = 0.80 t + 0.20, then a(t) = 0.80 m/s². At t = 2 s, x = 2.00 m and v = 1.80 m/s. Constant acceleration does not mean constant velocity; it means that velocity changes at a steady rate.

5. A force balance belongs to a system

For a constant-mass system in an inertial frame, Newton's second law has the vector form: sum of external forces = m × a. Along a chosen axis, project this relationship using consistent signs. Add the forces acting on the system, not the forces it exerts on its surroundings.

A trolley of mass 2.0 kg experiences a horizontal pulling force of 3.0 N and opposing friction of 1.0 N. The horizontal resultant is 2.0 N, so the acceleration is 1.0 m/s² in the direction of the pull. If pulling force and friction were equal, acceleration would be zero. The trolley could nevertheless continue moving at constant

velocity. “It is moving forwards, so the resultant is positive” is an incorrect conclusion.

The reciprocal forces of an interaction act on two different systems. They do not cancel in the force balance of either system alone. Similarly, weight and the ground's normal reaction can balance in a particular situation without being an action-reaction pair: their interaction partners are not the same.

6. Change the approach by using energy

Dynamics links forces and acceleration; an energy approach can avoid describing every instant of motion. For an object of mass m moving with speed v, kinetic energy is $E_c = (1/2)\, m\, v^2$. In a gravitational field assumed uniform, a potential energy can be written $E_p = m\, g\, h$, with a chosen height origin.

Conservation of mechanical energy requires appropriate assumptions. With friction, mechanical energy may decrease while total energy, including exchanges, remains accounted for. Transformation into internal energy is not the disappearance of energy. Specify the system boundary and the transfers crossing it.

An object of 2.0 kg whose speed changes from 5.0 to 10.0 m/s has its kinetic energy increase from 25 to 100 J. Doubling the speed quadruples this energy in the model. This does not permit predicting a real braking distance without knowing the forces, grip and conditions of motion.

7. Power is not an energy reserve

Average power is energy transferred per unit time: $P_{moy} = E / \Delta t$. One watt is one joule per second. A lamp operating at constant power of 5 W for two hours receives 10 Wh, or 36 000 J. Energy expressed in Wh and power expressed in W are therefore not interchangeable.

A battery rated at 20 Wh could power this lamp for four hours in an ideal model without losses. A real calculation must account for efficiency, temperature, operating limits and the energy actually usable. The ideal value gives an order of magnitude, not a promised operating time. No battery assembly is needed to understand this reasoning.

8. Discuss two models without inventing a scientific disagreement

We can compare a frictionless model with a model including friction. The first simplifies calculation and sometimes describes a short phase adequately; the second may better reproduce a longer evolution. The question is not which is always “true”, but which answers the question with an acceptable approximation.

A difference between measurement and prediction calls for several checks: calculation, units, procedure, uncertainty and assumptions. It does not immediately prove that the law studied is false. Conversely, saving a model by endlessly adding uncheckable assumptions removes its explanatory value. A useful explanation states the conditions in which it might fail.

9. Worked exercises

Exercise A - Signs. An object moves in the positive x direction at 4 m/s. Its velocity becomes 1 m/s after 3 s. Calculate the average acceleration. **Answer:** $(1 - 4)/3 = -1$ m/s². The sign describes the direction of the velocity change; it does not mean that the object is already moving in the negative x direction. Its final velocity remains positive.

Exercise B - Energy. An object of 0.50 kg moves at 6.0 m/s. What is its kinetic energy? **Answer:** $E_c = 0.5 \times 0.50 \times 6.0^2 = 9.0$ J. The two

factors 0.5 and 0.50 are respectively the formula's coefficient and the mass: confusing them is a common mistake.

Exercise C - Model. A lamp of 4 W operates for three hours. A battery is rated at 15 Wh, of which 80 % is assumed usable. **Answer:** required energy = 12 Wh; usable energy = 0.80 × 15 = 12 Wh. The model gives exactly the required amount, without a margin. It therefore does not justify guaranteed operating time under real conditions.

10. Active revision

Without looking at the text, define quantity, unit, frame of reference and resultant. Then explain why non-zero velocity can accompany zero acceleration, and why mechanical energy can decrease without total energy disappearing. Repeat a calculation while keeping the units, and identify an assumption that limits its scope.

For an oral answer, give an idea, a calculation or example, then a limitation. In writing, conclude in the terms of the initial question. An isolated numerical result shows neither what was studied nor why the calculation is appropriate.

11. Choosing a model from a graph

A table of measurements does not arrive with its equation. To decide whether a relationship is proportional, first examine what the origin represents. If an extension x is measured relative to a spring's unloaded length, a law F = kx passes through the origin in the linear elastic model. If the instrument has a constant offset, the data may instead follow F = kx + b. The term b is not automatically a new property of the spring: it may describe a zero error or a force already present. The distinction must be examined through the procedure, not chosen merely because a straight line looks attractive.

Consider the following fictional points: x is 0.010; 0.020; 0.030 m, and F is respectively 0.23; 0.43; 0.63 N. Between successive points, force changes by 0.20 N for an extension change of 0.010 m. The gradient is therefore 20 N/m. The linear relationship passing through these points is F = 20x + 0.03. Interpreting the gradient as stiffness requires a well-defined extension and an appropriate linear regime. Interpreting the intercept as an instrument error requires another check. The calculation distinguishes two parameters; it does not decide their causes on its own.

To test a relationship $F = kv^2$, plotting F against v^2 may make the structure more visible than plotting F against v. Transforming the axes puts a question to the data. It does not remove the need to check units and the range over which the assumption holds. Extrapolating a law fitted over a small range may produce a numerically very precise yet physically very fragile prediction.

12. A differential estimate of uncertainty

Suppose a velocity is calculated as v = d/t from independent measurements of distance and duration. A usual estimate of combined standard uncertainty, when relative uncertainties are small and the linearised model is appropriate, gives u(v)/v = sqrt[(u(d)/d)² + (u(t)/t)²]. This relationship combines standard uncertainties; it is not a sum of guaranteed maximum margins. Correlation between measurements would change the calculation. We therefore need to know the nature of the numbers used before inserting them into this expression.

With d = 2.00 m, u(d) = 0.01 m, t = 1.00 s and u(t) = 0.02 s, the calculated velocity is 2.00 m/s. The relative contributions are 0.005 and 0.020. Their combination in quadrature is about 0.0206, so the standard uncertainty in velocity is about 0.041 m/s. In this example, improving only the distance measurement will change overall uncertainty little: duration dominates. This conclusion directs a concrete experimental improvement rather than a multiplication of decimal places.

A more elementary approach can use given bounds: if d lies in [1.99; 2.01] m and t in [0.98; 1.02] s, then d/t lies in [1.99/1.02; 2.01/0.98], approximately [1.951; 2.051] m/s. This second calculation answers a different question and uses different assumptions. Presenting the approaches as identical would lose the information they carry. The intended skill is to choose and name the method, then judge which measurement deserves improvement.

Chapter 2 - Kinematics: describing motion without confusing its quantities

Historical reference point - 1638: combining two motions

Galileo's Discorsi, published in Leiden in 1638, discuss the parabolic motion of projectiles among other topics. The Museo Galileo connects this result with the combination of horizontal motion and uniformly accelerated falling motion. This historical reference point illuminates the separation of equations along two axes in this chapter. The drawn shape of a parabola does not prove that every real object will follow that trajectory exactly: air resistance and initial conditions must always be discussed. The history of an idea helps us understand why a demonstration must state its assumptions.

Reference for the historical fact, consulted for the French manuscript on 11 October 2026: Museo Galileo, catalogue entry for the apparatus demonstrating a parabolic trajectory, https://catalogue.museogalileo.it/object/ApparatusToDemonstrateParabolicTrajectoryProjectiles.html. LFDP educational analysis, without verbatim quotation or reproduced image.

1. The problem and the necessary choices

Kinematics describes where a system is and how its position changes. It does not yet seek the responsible force. This separation is productive: observing a trajectory allows us to formulate a dynamics question, but a trajectory alone does not reveal every interaction. A ball tracing a parabola in a video does not prove, solely through its shape, that friction is negligible. Relate this assumption to the conditions and the precision of the data.

Before calculating, name the system, the frame of reference, the spatial coordinate system and the time origin. The system may be an object whose centre of mass is studied, a useful approximation

when rotation is not the question. The frame of reference is the physical framework relative to which motion is described. The coordinate system provides axes, an origin and coordinates. The time origin defines what t = 0 means: the start of a recording, a launch or passing a marker. These choices are not presentation details; they fix the meaning of the quantities.

A train moving steadily illustrates dependence on the frame of reference. A seated passenger has a constant position in the carriage's coordinate system. In the platform's, their position changes. Two different descriptions can be correct because they answer different questions. An Earth-fixed frame will generally be used here; its approximation as an inertial frame will be discussed in the chapter on forces. To begin, all coordinates are expressed in a fixed orthonormal coordinate system, and time in seconds.

2. Position, trajectory and displacement

In two dimensions, the position vector is OM(t) = x(t) e_x + y(t) e_y. The vectors e_x and e_y represent the axis directions. The trajectory is the set of positions occupied; the position-time functions additionally state when they are occupied. Two objects can follow the same curve at different rates. Looking only at a photograph of this curve therefore does not establish velocity.

Displacement between two times is the difference between the position vectors. It depends on the journey's endpoints, whereas distance travelled depends on the entire path. A walker travels 60 m east, then 40 m west. The total distance is 100 m; displacement is 20 m east. If the journey takes 50 s, average velocity is 0.40 m/s east, whereas average speed, meaning distance divided by duration, is 2.0 m/s. Specify which meaning is used to avoid ambiguity.

Straight-line motion occurs along a straight line but can include a reversal of direction. A coordinate x(t) that increases and then decreases represents this reversal without requiring the axis to move. Circular motion is defined by a circular trajectory; it can be uniform or non-uniform. Here, “uniform” describes constant speed, the magnitude of velocity. The terminology thus combines a geometric property of the trajectory with a time-dependent property, which must be examined separately.

3. Velocity as a change in position

Average velocity between t and t + Δt is ΔOM/Δt. Instantaneous velocity is the limit of this quotient as Δt tends to zero, if position is differentiable. We write v = dOM/dt, hence $v_x = dx/dt$ and $v_y = dy/dt$. Its magnitude is $\sqrt{v_x^2 + v_y^2}$ in our coordinate system. Its units are metres per second. The velocity vector is tangent to the trajectory and points in the direction of motion when its magnitude is non-zero.

This definition connects graphical representation with calculation. On a graph of x against t, the gradient of a secant gives an average velocity. The tangent's gradient gives v_x at a particular time. A negative gradient means that the coordinate decreases; it does not mean that speed is negative. On a velocity-time graph, a signed area gives a displacement, not an absolute position without an initial condition.

Consider $x(t) = 3 + 2t - 0.50t^2$, with appropriate units. Then $v_x(t) = 2 - t$. At t = 1 s, the object moves in the positive direction; at t = 2 s, its velocity is zero; at t = 3 s, it moves in the negative direction. Its coordinate at t = 2 s is 5 m. Zero velocity at that instant erases neither its position nor its later evolution. Here it characterises a reversal, established by the change in sign of v_x around that time.

4. Acceleration describes a vector change

Acceleration is a = dv/dt. Its components are the derivatives of the velocity components, and its units are m/s². It can change the magnitude of velocity, its direction or both. In the previous example, a_x = −1 m/s². Before the reversal, velocity and acceleration point in opposite directions: speed decreases. After the reversal, they point in the same direction: speed increases. The same negative acceleration therefore corresponds to two different changes in speed.

For straight-line motion, comparing the signs of v_x and a_x is useful. A positive product indicates increasing speed, except at the point where velocity is zero; a negative product indicates decreasing speed. Do not transfer this rule unchanged to motion in a plane by arbitrarily comparing just one component. We must instead consider the projection of acceleration onto the direction of velocity.

Zero acceleration over an interval implies a constant velocity vector over that interval. Zero acceleration at a single time is insufficient. Similarly, constant speed does not imply zero acceleration. Uniform circular motion provides a decisive counterexample: the velocity vector rotates, and its derivative points towards the centre. The everyday intuition that “accelerating” means “going faster” must therefore be enriched to become a physical concept.

5. Constructing position-time equations for constant acceleration

Along an axis, if $a_x = a$ is constant and $v_x(0) = v_0$, integration gives $v_x(t) = v_0 + at$. A second integration, with $x(0) = x_0$, gives $x(t) = x_0 + v_0t + at^2/2$. Integration constants carry the initial conditions. Forgetting them means solving a different problem. The expression applies only during the phase in which acceleration is assumed constant.

We can recover displacement from the velocity graph: the area of a rectangle v_0t plus the signed area of a triangle $at^2/2$. This second approach is not a competing formula; it illuminates why time appears squared. To compare two trajectories, keep the same origins or state their differences. Two expressions containing different constants are not necessarily incompatible.

A moving object starts at $x_0 = 10$ m with $v_0 = 4$ m/s and $a = 2$ m/s². After 3 s, its velocity is 10 m/s and its position is 31 m. Displacement is 21 m. Average velocity during this phase is $(4 + 10)/2 = 7$ m/s, because velocity varies linearly; multiplying by 3 s gives the same displacement. Using the final velocity for the whole duration would give 30 m and overestimate displacement.

6. A projectile in a uniform field: an explicit model

Take a projectile launched from the origin with initial speed v_0 at an angle α above the horizontal. In the model without air resistance, acceleration is $(0, -g)$, with constant g. Initial conditions give $x(t) = v_0 \cos(\alpha)t$ and $y(t) = v_0 \sin(\alpha)t - gt^2/2$. Horizontal and vertical motions share the same time variable; studying them separately does not make them physically independent.

If $\cos(\alpha)$ is non-zero, eliminate t to obtain $y = x \tan(\alpha) - gx^2/[2v_0^2 \cos^2(\alpha)]$. The trajectory is a parabola in this model. At the highest

point, $v_y = 0$, but horizontal velocity generally remains non-zero. The time to reach it is $v_0 \sin(\alpha)/g$. For a return to the same height, the positive flight time is twice this value, and the range is $v_0^2 \sin(2\alpha)/g$. This last expression does not apply directly if the starting and finishing heights differ.

With $v_0 = 10$ m/s, $\alpha = 30°$ and $g = 10$ m/s² chosen for the exercise, we obtain $v_y(0) = 5$ m/s, a highest point at 0.50 s and a maximum height of 1.25 m. Return to the initial level occurs at 1.0 s; the range is about 8.66 m. These numbers describe an ideal fictional situation, not instructions for an actual launch. Wind, object shape and air resistance can substantially alter a trajectory.

7. Circular motion: separating direction and speed

A circular position can be parameterised by $x = R\cos(\omega t)$ and $y = R\sin(\omega t)$, with constant R and ω. Differentiation gives $v_x = -R\omega \sin(\omega t)$, $v_y = R\omega \cos(\omega t)$, then a magnitude $R|\omega|$. A second differentiation gives $a_x = -R\omega^2 \cos(\omega t)$, $a_y = -R\omega^2 \sin(\omega t)$. Acceleration is therefore opposite to the position vector: it points towards the centre, and its magnitude is $R\omega^2 = v^2/R$.

The period is $T = 2\pi/|\omega|$. On a circle of radius 2 m completed in 4 s, speed is $2\pi R/T = \pi$ m/s and the magnitude of acceleration is $\pi^2/2$, about 4.93 m/s². The velocity vector is never constant during the circuit, although its magnitude is. This acceleration has a kinematic definition; the next chapter will seek the resultant force that can produce it.

When speed changes, a tangential component is added to the normal component. In the local coordinate system of the trajectory, $a_t = dv/dt$ for speed v, and $a_n = v^2/R$ for a circular trajectory. The local coordinate system rotates with the motion: its basis vectors must not be differentiated as if fixed. This

clarification avoids confusing a convenient representation of acceleration with the correct calculation of its derivative.

8. Estimating velocity from a recording

Positions measured at regular intervals Δt can provide a central estimate $v(t_i) \approx [x_{(i+1)} - x_{(i-1)}]/(2\Delta t)$. This expression uses neighbouring points; it does not directly give values at the recording's endpoints. Decreasing Δt may better describe rapid changes, but position noise becomes more important relative to the calculated difference. Finer sampling therefore does not automatically improve every estimate.

For x at three successive times equal to 0.12; 0.20; 0.31 m, with Δt = 0.10 s, the estimated central velocity is (0.31 − 0.12)/0.20 = 0.95 m/s. Average velocities over the two intervals are 0.80 and 1.10 m/s. Their evolution suggests positive acceleration in this frame. With only three uncertain positions, it does not prove that acceleration remains constant throughout the trajectory.

Acceleration estimated by a second difference is $[x_{(i+1)} - 2x_i + x_{(i-1)}]/\Delta t^2$. Here it is 3.0 m/s². The squared denominator makes the estimate sensitive to position errors. Before interpreting a sudden oscillation in acceleration, examine resolution, calibration and point tracking. A calculated curve is not a direct observation independent of the data that produced it.

9. Worked exercises and errors to recognise

A - Return journey. A moving object travels 30 m east in 10 s, then returns 10 m west in 5 s. Calculate distance, displacement and the two averages of motion. **Answer:** distance 40 m, displacement 20 m east, duration 15 s. Average velocity is 20/15 = 1.33 m/s east; distance/duration is 2.67 m/s. The simple mean of 3 and 2 m/s is inappropriate: the phases have different durations.

B - Ideal stop. A moving object has $v_0 = 6$ m/s and $a_x = -2$ m/s² until its first stop. **Answer:** $v_x = 6 - 2t$ reaches zero at 3 s. Displacement is $6\times3 - 3^2 = 9$ m. Extending this expression beyond 3 s would predict reverse motion; this extension is valid only if the same acceleration actually remains applied. For simple braking that ends at rest, change the phase and model.

C - Highest point. Is acceleration zero at the top of an ideal parabolic trajectory? **Answer:** no, it is still $(0, -g)$. Only the vertical velocity component is zero. The confusion arises from replacing the derivative of velocity with velocity itself.

D - Reading a graph. A curve $x(t)$ is a straight line with gradient -3 m/s. **Answer:** motion is uniform and straight in this frame, with $v_x = -3$ m/s and $a_x = 0$. A different curve $v_x(t)$ with gradient -3 m/s² would describe constant acceleration, a different situation. Axis labels determine the gradient's meaning.

10. Revision and scope

For revision, reconstruct the position-time equations from acceleration and initial conditions, then check them by differentiation. Draw position, velocity and acceleration separately for a reversal. Explain circular motion without saying “velocity does not change”: say its magnitude remains constant while its direction changes. Every description must specify the frame of reference, units and phase studied.

This chapter develops original LFDP reasoning and exercises. Its framework is the published official physics and chemistry curriculum, physics component: https://www.education.gouv.fr/bo/19/Special8/MENE1921249A.htm ; appendix consulted for the French manuscript on 11 October 2026: https://cache.media.education.gouv.fr/file/SPE8_MENJ_25_7_2019/92/9/spe249_annexe_1158929.pdf. No curriculum text, examination question or third-party diagram is reproduced. Numerical situations are fictional and must not serve as road-safety instructions or instructions for dangerous experiments.

Chapter 3 - Dynamics: connecting a force balance with acceleration

Historical reference point - 1687: organising mechanics through principles

Newton's Principia was published in 1687 under the auspices of the Royal Society, which holds the manuscript associated with the first edition. For the reader, this milestone places the laws studied within a mathematical construction of mechanics, rather than a simple list of numerical recipes. This chapter's approach remains one of justification: choose the system, identify interactions, project and then solve. A work's historical importance does not remove the need to state the limits of the Newtonian framework when a situation requires a different model.

Reference for the historical fact, consulted for the French manuscript on 11 October 2026: Royal Society, account of the Principia manuscript, https://royalsociety.org/blog/2014/07/principia/. LFDP educational analysis, without verbatim quotation or reproduced image.

1. Define what receives the forces

A force describes an interaction exerted on a system by another system. Before listing forces, choose the boundary studied. For a book resting on a table, the system can be the book alone: Earth and the table then exert external forces. If book and table are studied together, their mutual interaction becomes internal. A force list therefore cannot be correct independently of the system chosen. This is the first decision in a dynamics exercise.

Represent each force by a vector, characterised by its direction, orientation and magnitude in newtons. For the translational motion of the centre of mass, the external resultant is decisive. Rotation problems require points of application and moments; this guide

does not replace a complete course in rigid-body mechanics. Drawing arrows towards a point representing the centre of mass is a diagram convention, not a claim that all interactions physically occur there.

List interaction partners before seeking formulas: Earth, support, cable, air, another charged object. Motion is not a partner. There is therefore no “forward-motion” force to add because an object moves. The velocity vector describes the state of motion; forces describe interactions capable of changing momentum.

2. Inertia and inertial frames

In an inertial frame, a system that is isolated or subject to zero external resultant retains a constant velocity vector. It can be at rest or in uniform straight-line motion. Inertia is therefore not a special tendency to stop. The stops observed in everyday life often result from interactions with the ground or air. Their ubiquity explains the mistaken intuition that a force is essential to maintain any velocity.

A frame in uniform straight-line translation relative to an inertial frame is itself inertial in classical mechanics. A coordinate system attached to an accelerating vehicle is not, under the same conditions. Describing a free ball as accelerating relative to that vehicle does not mean that a new physical interaction was applied to it. To use the second law in its simple form, choose an appropriate frame; otherwise, inertial terms must be introduced within an explicitly stated theory.

An Earth-fixed frame is a usual approximation for the modest durations and distances of a school exercise. Earth's rotation limits this approximation, importantly for some extended or very precise motions. An assumption can be excellent for a trolley tracked for two seconds and insufficient for atmospheric circulation. A frame's

suitability depends on the problem, not an absolute label of “true” or “false”.

3. The second law and projection

Momentum is p = mv. The usual general form for a material system in an inertial frame connects the external resultant to dp/dt. For a closed system of constant mass, this becomes sum F_ext = ma. Variable-mass problems, such as an open system ejecting matter, require an appropriate balance; simply using ma with changing mass without examining exchanges is insufficient.

In a fixed orthonormal coordinate system, the vector relationship gives an equation for each axis: sum F_x = ma_x, sum F_y = ma_y. Signs arise from projection, not the force's name. An object's weight along an upward vertical axis is −mg; along a downward axis, it is +mg. Friction opposes relative sliding in a simple model, but its sign depends on the direction of sliding and the chosen axis.

The method has six steps: state the system and frame; identify interactions; choose axes; project forces; solve for acceleration or the requested unknown; interpret. If motion is constrained to a support, this supplies an additional equation. We cannot always assume N = mg before projecting: an angled pull, an inclined plane or vertical acceleration would change the normal reaction.

4. Weight, mass and contact

Mass characterises inertia in the relationship ma, among other properties; weight is the gravitational force exerted by Earth in the near-surface model P = mg. A given mass retains its value when changing location in this guide's ordinary situations, whereas weight depends on the gravitational field. The kilogram is a unit of mass; the newton a unit of force. Saying a weight is “2 kg” mixes two quantities, even if everyday language uses this expression.

A support's contact force can be resolved into a normal component and a tangential component. On a horizontal support without vertical acceleration, an object subject only to weight and contact satisfies N = mg. This does not mean that weight and reaction form an action-reaction pair: weight is exerted by Earth on the object, and reaction by the support on the object. The corresponding reciprocal forces are exerted by the object on Earth and on the support.

A lift provides a fictional example. For a person of mass 60 kg, with g = 10 m/s² for calculation, upward acceleration of 1 m/s² gives N − mg = ma, hence N = 660 N. With downward acceleration of the same magnitude, N = 540 N. Mass has not changed; contact force has. A weighing scale based on this contact force can therefore display a different value when used outside the conditions assumed for its conversion.

5. Friction: a relationship depending on the regime

Static friction can prevent sliding and adjusts, in a simple model, up to a maximum magnitude μ_s N. While the object does not slide, automatically imposing f = μ_s N is incorrect. If a horizontal pulling force of 2 N is balanced by possible static friction of that magnitude, the horizontal resultant is zero. The maximum is a bound, not a value always reached.

Once sliding occurs, a simplified solid-friction model uses f = μ_k N, directed against velocity relative to the support. Coefficients depend on the surfaces and conditions; they are not guaranteed universal constants. Air resistance follows other models, such as dependence on speed or its square in certain ranges. Confusing these regimes would misidentify the mechanism, even if the calculation's units remained consistent.

The orientation is not always “backwards” in the Earth-fixed frame. A crate carried without sliding by an accelerating conveyor can receive a frictional force in the direction of its acceleration. The correct question is which relative sliding would be prevented or opposed. This example shows why both surfaces and their relative motion should be defined before drawing the arrow.

6. An inclined plane: using coordinate geometry

Take an object of mass m on a plane inclined at angle α. Choose an axis parallel to the plane, directed downwards, and an outward normal axis. Weight has a component mg sin(α) down the slope and −mg cos(α) along the normal. If the object remains in contact with the plane and has no normal acceleration, the normal reaction is N = mg cos(α). Geometry is simpler in this coordinate system than with horizontal and vertical axes.

Without friction, acceleration down the slope is g sin(α), independent of mass in this model. With sliding friction opposing descent, it is g[sin(α) − μ_k cos(α)]. A negative value would mean that downward speed decreases during this phase; it would not prove that the system spontaneously begins moving upwards after stopping. At rest, the static regime and its conditions must be examined.

In a fictional exercise with m = 2 kg, α = 30°, μ_k = 0.20 and g = 10 m/s², N is about 17.32 N. Friction is about 3.46 N and the weight component down the slope is 10 N. The resultant is therefore about 6.54 N, giving a ≈ 3.27 m/s² downwards. Checking the order of magnitude is useful: friction here reduces acceleration compared with the frictionless model's 5 m/s². This example yields no real surface coefficient.

7. Tension and a combined system

An ideal rope, with negligible mass and no extension, transmits tension under appropriate assumptions. In an arrangement without a massive pulley or friction, the tension magnitude can often be treated as identical along one rope segment. This simplification belongs to the model; it does not automatically apply to a massive or elastic rope, or one passing over an energy-dissipating mechanism.

Two fictional trolleys, of masses 1 kg and 2 kg, are connected and pulled horizontally by a force of 6 N, without friction. Taking both as the system, the tension between them is internal: acceleration is $6/(1 + 2) = 2$ m/s². If the force pulls the trolley of 1 kg, the second receives only tension horizontally; $T = 2\times2 = 4$ N. For the first, $6 - T = 1\times2$ recovers the same result.

This double study illustrates a strategy: the combined system simplifies calculation of a shared acceleration, then a subsystem reveals an internal interaction. Adding all forces from both diagrams without distinguishing the systems would count tension twice. Choosing an appropriate boundary is therefore a solution method, not a formality before the “real calculation”.

8. A resultant perpendicular to velocity

For uniform circular motion, kinematics gives an acceleration towards the centre of magnitude v^2/R. Dynamics then requires a radial resultant of magnitude mv^2/R. The expression “centripetal force” names this resultant's role; it does not add an interaction to forces already present. Depending on the situation, tension, gravitation or a combination of contact forces may fulfil this role.

An object of 0.20 kg follows an ideal horizontal circle of radius 0.50 m at 2.0 m/s. The resultant towards the centre must be $0.20\times2.0^2/0.50 = 1.6$ N. If tension is the only horizontal force, its

horizontal component has this magnitude. Vertical equilibrium still needs to be addressed. The question must specify the arrangement; radial calculation alone does not establish that total tension is 1.6 N in every configuration.

A force perpendicular to velocity can change its direction without instantaneously changing its magnitude. This prepares the connection with a force's power, defined later by the scalar product F·v. Descriptions through forces and through energy do not contradict one another: they examine different aspects of the same evolution. A force that does no work does not necessarily have no dynamical effect.

9. Electric field and the charge's sign

In a uniform electric field E, a particle of charge q receives electric force F = qE. For positive charge, this force points with the field; for negative charge, it points oppositely. Assuming a single relevant force and constant mass, acceleration is a = qE/m. A particle already moving does not necessarily travel in the direction of that acceleration: initial conditions remain essential.

For a fictional particle of mass 2.0×10^−6 kg and charge 3.0×10^−9 C, a field of 4.0×10^3 N/C gives a force of 1.2×10^−5 N and acceleration of 6.0 m/s². With the opposite charge, only this acceleration's orientation changes. In a coordinate system where the field points along positive x, a_x therefore takes the sign of q. The units N/C multiplied by C do indeed give N.

This theoretical example describes neither an electrical experiment nor a household device. It shows how a field translates an interaction distributed in space into a force on a test object. For a very light real particle, weight, collisions or relativistic effects must be examined according to conditions; they are not eliminated simply by writing F = qE.

10. Worked exercises

A - Angled force. A block of 2 kg receives a pulling force of 5 N at 30° above the horizontal. It remains on a horizontal support, without friction, with g = 10 m/s². **Answer:** vertically, N + 5 sin(30°) − 20 = 0, hence N = 17.5 N. Horizontally, a_x = 5 cos(30°)/2 ≈ 2.17 m/s². Imposing N = 20 N would ignore the pull's vertical component.

B - Zero resultant. A trolley moves at 3 m/s with two opposing horizontal forces of 4 N. **Answer:** their sum is zero; in the constant-mass inertial model, velocity remains constant. Rest is not the only solution. If one force disappears, the phase changes and acceleration becomes non-zero.

C - Reciprocal interaction. Two objects repel each other. One is more massive. Are the reciprocal force magnitudes different? **Answer:** they are equal and opposite in the mechanics studied. Accelerations can differ because F/m depends on mass. Equal forces do not impose equal effects on two different systems.

D - Impossible contact. A calculation gives N < 0 for a support that can only push. **Answer:** the permanent-contact model has become incompatible with the situation. A negative normal reaction is not a physical value to preserve by taking its absolute value. Examine a phase of contact loss with a new balance and new conditions.

11. Revision and references

Reconstruct a force balance without looking at the worked diagram, then check that each force has an interaction partner. Explain why the state of motion is not a force, why static friction is not always at its limit and why a centripetal resultant is not an additional interaction. For every projection, justify the sign before replacing symbols with values.

Framework for progression: published official final-year curriculum, physics component, https://www.education.gouv.fr/bo/19/Special8/MENE1921249A.htm. Reference points on Newton's laws: NASA Glenn, https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/newtons-laws-of-motion/. The reasoning, examples and exercises above are LFDP originals; no third-party passage, diagram or examination question is reproduced. The stated assumptions belong to educational situations and certify no real arrangement.

Chapter 4 - Energy and work: choosing a boundary and tracking transfers

Historical reference point - 1849: linking mechanical work and heat transfer

The Royal Society's archives hold a manuscript by James Prescott Joule dated 4 June 1849, concerning the mechanical equivalent of heat; it was received on 6 June and presented on 21 June. This document provides a concrete reference point in studying relationships between work and heat transfer. It does not mean that the entire first law was already contained in one school-level formula. In this guide, its educational role is to remind us that decreasing mechanical energy should lead us to investigate transfers and other energy forms, rather than conclude that energy has disappeared.

Reference for the historical fact, consulted for the French manuscript on 11 October 2026: Royal Society, archives PT/37/3, https://makingscience.royalsociety.org/items/pt_37_3/paper-on-the-mechanical-equivalent-of-heat-by-james-p-prescott-joule. LFDP educational analysis, without verbatim quotation or reproduced image.

1. Why an energy approach?

Dynamics allows us to find acceleration, then a trajectory if initial conditions are known. Some questions ask only for a speed after a displacement or an energy transfer. An energy approach can then avoid calculating every instant of motion. It does not replace forces in every situation: kinetic energy depends on speed and does not establish velocity's direction on its own.

Take a ball descending an ideal track. A differential equation along the track could describe its detailed motion. If we seek only its speed at a given height, an energy balance can be much more

direct. However, knowing that speed does not tell us whether the ball remains in contact in a bend. This latter question requires a projection of Newton's second law. Choosing a method therefore means examining what information it retains and what it does not provide.

The first precaution is to define the system. An object alone, an object together with Earth and an object together with its support do not have the same list of external transfers. Decreasing mechanical energy of an object does not mean a loss of total energy from everything around it. The word “loss” often refers to energy becoming less useful for the intended purpose, for example through transformation into internal energy, rather than physically disappearing.

2. Work done by a constant force

For a constant force F applied at a point moving from A to B, work is W_AB(F) = F·AB = F AB cos(θ), where θ is the angle between force and displacement. Work is a signed quantity, expressed in joules. It is positive if the force has a component with the displacement, negative if that component opposes it, and zero if perpendicular.

A constant pulling force of 10 N makes an angle of 30° with a horizontal displacement of 3 m. Its work is 10×3 cos(30°), about 26.0 J. The same force magnitude in a vertical direction would do no work for this horizontal displacement. Adding force magnitudes alone would therefore be insufficient: orientation is essential. The scalar product expresses precisely this dependence.

For a variable force, add infinitesimal amounts of work F·dr along the path; in one dimension, this becomes an integral of F(x) dx. A signed area under a graph of F against x can represent this work. A region below the axis contributes negatively. Do not replace the

integral by “final force times distance” unless an assumption permits this shortcut.

3. The work-kinetic energy theorem

A constant-mass particle's kinetic energy is $E_c = mv^2/2$, where v denotes speed. For this model in an inertial frame, the kinetic energy change between A and B equals the sum of work done by external forces. Writing $\Delta E_c = \sum W$ does not assume a constant resultant. However, the forces must belong to the system studied, and their work must be evaluated over the relevant displacement.

We understand the relationship's origin by taking the scalar product of $ma = \sum F$ with velocity v. Since $d(v\cdot v)/dt = 2a\cdot v$, we obtain $d(mv^2/2)/dt = \sum F\cdot v$. Integrating over time connects energy change with work. This demonstration uses constant mass and a particle model. A rotating solid also has rotational kinetic energy; reducing it without justification to its centre-of-mass kinetic energy would be incomplete.

A trolley of 2 kg changes speed from 1 to 3 m/s. Its kinetic energy change is $(2/2)(9 - 1) = 8$ J. If a pull supplies 12 J and another force supplies −4 J, the sum is indeed 8 J. The theorem does not say that the pull alone did work. It can also determine work done by an unknown force when the other transfers and speeds are known.

4. Conservative forces and potential energy

A conservative force allows a potential energy to be defined whose change is the opposite of its work: $W_{AB} = -[E_p(B) - E_p(A)]$. Its work depends only on the endpoints within the domain considered. Potential energy is defined only up to an additive constant. Choose a convenient origin; changes and physical results do not depend on it.

In a uniform gravitational field with an upward vertical axis z, $E_p = mgz + \text{constant}$. Rising through height h increases this energy by mgh, and work done by weight is $-mgh$. Descending gives the opposite sign. The formula includes the interaction with Earth; it must not be understood as a mysterious reserve attached to an absolute height without a reference.

For an ideal spring of stiffness k and extension x measured from its unloaded state, restoring force is $F_x = -kx$. An appropriate antiderivative gives $E_p = kx^2/2$. This is positive for both extension and compression, with zero in the unloaded state. The force points towards that state in both cases. This form assumes a linear elastic range: it does not predict the behaviour of a heavily deformed or damaged spring.

5. Conservation and the mechanical energy balance

Mechanical energy is $E_m = E_c + E_p$ for the conservative interactions included. If only these forces do work, E_m remains constant in the model. If non-conservative forces do work, the mechanical energy change corresponds to their work, when the system and its interactions have been correctly defined. Friction on a stationary support can thus reduce E_m.

A fictional ball of mass 0.50 kg starts from rest 3.0 m above its arrival level. With $g = 10\ m/s^2$ and no rotation or friction, initial energy is 15 J. At arrival it is entirely kinetic: $v = \sqrt{2gh} = \sqrt{60}$, about 7.75 m/s. Mass cancels because it multiplies both energies. This simplification does not mean that all real objects descend identically through air.

Now suppose friction does work of −3.2 J during descent. Final kinetic energy is 11.8 J, hence $v = \sqrt{2 \times 11.8/0.50} \approx 6.87$ m/s. Final mechanical energy is lower; total energy has not disappeared. To describe where it has been transferred, expand

the system to include the support and thermal exchanges. If the ball rolls, part of its kinetic energy is rotational: that would be another model again.

6. Ideal stopping distance and limits on its use

A theoretical braking model with a constant resisting force of magnitude f on a fixed support gives $W = -fd$. If initial speed is v_0 and final speed zero, the work-kinetic energy theorem leads to $d = mv_0^2/(2f)$. If $f = \mu mg$ in a sliding model, the distance becomes $v_0^2/(2\mu g)$. Mass cancels only because the chosen force is proportional to it.

Within the model, this expression shows a quadratic dependence on initial speed. Doubling v_0 quadruples d if μ and g remain identical. It provides no guaranteed real distance: reaction time, support condition, slope, braking control, variations in force and interaction with air are not described. A school result must not become operational driving guidance.

Distance travelled during a preceding constant-speed phase is $v_0\Delta t$. It can be added to a braking distance if the phases are clearly separated and the model justifies this. Such addition illustrates a good habit: identify time boundaries before combining balances. Using one formula with quantities belonging to different phases often produces a meaningless answer.

7. A spring and exchanges between two energy forms

An object of mass 0.25 kg is attached to an ideal spring of stiffness 100 N/m. In a horizontal model without friction or other work, an initial extension of 0.20 m at rest corresponds to potential energy of 2.0 J. Passing through equilibrium, the chosen potential energy is zero; kinetic energy is 2.0 J, giving $v = \sqrt{2\times2.0/0.25} = 4.0$ m/s.

The energy approach gives this speed without calculating the time of passage. Direction depends on the phase of motion. A dynamics study would give an oscillator equation and allow calculation of the period. The methods complement one another: one describes an overall constraint, the other evolution over time. It is useful to know what is missing before believing the exercise entirely solved.

On a potential energy graph, a minimum corresponds to stable equilibrium for this ideal one-dimensional system: a small displacement produces a restoring force. A maximum corresponds to unstable equilibrium. Saying force is zero at a point therefore does not establish stability. The energy's shape around that point supplies the necessary information. Friction can subsequently damp motion without changing this local distinction in the conservative model.

8. Power, efficiency and a conversion chain

The instantaneous power of a force applied at a point with velocity v is P = F·v. Its average over a duration is work divided by that duration. One watt is one joule per second. High power can supply a given energy rapidly; it is not total energy. A reserve expressed in Wh and power expressed in W must therefore not be added or compared as identical quantities.

A fictional mechanism supplies useful work of 600 J in 15 s. Its average useful power is 40 W. With an energy efficiency of 0.85 for this phase, energy received is 600/0.85 ≈ 706 J. Efficiency is useful energy divided by energy received, for a defined boundary and purpose. It does not mean that the remaining energy was annihilated: it corresponds to transfers that do not serve the chosen purpose.

For two successive conversions with efficiencies of 0.80 and 0.90, overall efficiency is 0.72 if the definitions and phases are

compatible. It is not the mean, 0.85. This can be understood using 100 J received: the first stage gives 80 useful to the second, which supplies 72. Relative losses apply to different amounts. Recovering part of the transfers may improve a chain, but does not bypass energy conservation.

9. Choosing between forces and energy

Use forces if the question concerns acceleration, direction, a support reaction or tension. Use energy if work is easily calculated and the question concerns a speed, height or transfer. Often both are needed. On a circular trajectory, energy can determine speed at a height; radial projection then determines the necessary contact force.

A particular difficulty arises when a contact force's work depends on support motion. Friction does not always do negative work on every object in every frame. A conveyor carrying a crate can supply it with energy. Overall dissipation must be studied by considering both systems and their relative motion. This nuance prevents a frequent property of friction on fixed ground from becoming a misapplied absolute rule.

Energy calculation also requires a consistent frame: kinetic energy depends on velocity in the chosen frame. An object does not have the same kinetic energy for an observer accompanying it as for one on the ground. Balances remain consistent if all quantities are defined in the same framework. Mixing velocities from different frames in one equation does not produce a usable conservation relationship.

10. Worked exercises

A - Zero work. A force of 8 N remains perpendicular to a displacement of 5 m. **Answer:** work is zero, because cos(90°) = 0. This force can nevertheless change velocity's direction. Zero work therefore does not establish zero acceleration.

B - Height. A particle of 0.20 kg has kinetic energy of 4.0 J before an ideal frictionless ascent. With g = 10 m/s², through what height increase can it rise if all this energy becomes potential? **Answer:** mgh = 4.0 gives h = 2.0 m. Assume that no other energy form or constraint alters the balance. The chosen height zero does not change this difference.

C - Variable force. Along an axis, a force is F_x(x) = 6x, with x in metres and the coefficient in N/m. Calculate its work from x = 0 to x = 2 m. **Answer:** integrating 6x gives 3x²; work is 12 J. Using the final force of 12 N over 2 m would give 24 J, because the force did not have its final value throughout the path.

D - Efficiency. A chain receives 500 J, transmits 400 to a second stage and finally supplies 300 J of useful energy. **Answer:** efficiencies are 400/500 = 0.80 and 300/400 = 0.75; their product, 0.60, equals 300/500. The two non-useful transfers are each 100 J in this example, but their relative efficiencies differ.

E - Impossible conclusion. Two objects have the same kinetic energy. Do they have the same speed? **Answer:** not necessarily, because E_c also depends on mass. For energy of 10 J, a mass of 1 kg has speed sqrt(20); a mass of 4 kg has speed sqrt(5), half as great. Equality of one quantity does not isolate a parameter without knowing the others.

11. Revision and sources

For revision, write a symbolic balance before any numerical substitution. Justify the signs of work, name the interactions corresponding to each potential energy and state conservation conditions. Then explain what an energy result cannot determine. The aim is to choose an appropriate method rather than seek, at any cost, a formula into which all the data can fit.

Framework and unit references: French Ministry of National Education, published final-year curriculum, https://www.education.gouv.fr/bo/19/Special8/MENE1921249A.htm ; BIPM, SI Brochure, https://www.bipm.org/en/publications/si-brochure/, consulted for the French manuscript on 11 October 2026. The discussion, demonstrations and exercises are LFDP originals. The fictional situations in this chapter prescribe no electrical assembly, braking experiment or risky spring manipulation.

Chapter 5 - Gravitation and orbits: understanding continuous falling motion

1. An interaction at a distance and a model

Newtonian gravitation describes attraction between two masses. For two objects treated as points, of masses M and m separated by distance r, the force magnitude is GMm/r^2. The constant G is universal within this model; it must not be confused with g, which describes a local gravitational field. Both objects receive reciprocal forces of equal magnitude and opposite direction.

Treating objects as points requires justification. Outside a perfectly spherical body with a spherically symmetric mass distribution, its gravitational field is that of a mass concentrated at the centre. A real body may depart from this shape and distribution. School-level orbital calculations often adopt the approximation; precise space navigation may find it insufficient. A result gains no physical precision merely through using many digits in calculation.

Choose a unit vector e_r directed from the attracting centre towards the satellite. The force on the satellite is $F = -GMm\, e_r/r^2$. The minus sign expresses attraction. In a frame centred on the massive object, considered approximately fixed and inertial, satellite acceleration is $a = -GM\, e_r/r^2$. Mass m cancels in this relationship; it remains present in energy or force. The frame's conditions and absence of other interactions are assumptions to state.

2. From the gravitational field to weight near the ground

The gravitational field created by mass M at distance r has magnitude GM/r². Near a spherical surface of radius R, we can write g ≈ GM/R² if rotation and local peculiarities are neglected. At altitude h, distance from the centre is R + h, not h. Forgetting the attracting body's radius would make force diverge artificially as altitude tends to zero.

The ratio of the fields at altitude h and at the surface is [R/(R + h)]². For h = R, this ratio is 1/4. At an altitude small relative to R, the field varies little, and assuming uniform gravity may be appropriate. This approximation has a domain of validity; it must not be extended to distances comparable with the radius without examination.

In an exercise with R = 6.0×10^6 m and h = 3.0×10^5 m, the ratio is (6.0/6.3)² ≈ 0.907. The field is therefore about 9.3 % weaker than the model's surface field. This result describes the fictional data proposed, not a measurement of a real location. Above all, it shows why an altitude small on a human scale can produce a non-negligible change in an orbital calculation.

3. Circular orbit: reconciling tangential velocity and radial acceleration

For uniform circular motion of radius r, kinematics requires acceleration of magnitude v²/r towards the centre. With gravitation the only included force, equate v²/r and GM/r². This gives v = sqrt(GM/r). Velocity is tangential, whereas force and acceleration are radial. They neither oppose velocity nor slow the object in this model; they change its direction.

An orbit can be understood as a fall continuing around the attracting body. The object is not beyond gravity's action. An appropriate horizontal velocity and the field's curvature produce a

trajectory that does not meet the surface in the model. This explanation is a conceptual image; establishing a real orbit involves many more trajectory, propulsion and environmental constraints.

The period is T = 2πr/v, hence T = 2π sqrt[r³/(GM)]. Setting μ = GM allows calculation without separating the two parameters. For a fictional exercise with μ = 4.00×10^14 m³/s² and r = 8.00×10^6 m, speed is about 7.07×10^3 m/s. The period is about 7.11×10^3 s, or about 118 min. Checking dimensions in sqrt(μ/r) does indeed give m/s.

4. Comparing two orbits without repeating every calculation

Around the same attracting centre, v is proportional to r^−1/2 and T to r^3/2 for ideal circular orbits. If radius is multiplied by four, speed is divided by two and period multiplied by eight. Reasoning through ratios reduces the risk of error and reveals the laws' structure. A large orbit can be travelled more slowly while requiring much longer for one revolution.

Orbital radius and altitude are not interchangeable. If two altitudes have a ratio of four, their radii R + h generally do not. The preceding reasoning must therefore concern distances from the centre. Writing the symbol r and recalling its definition before calculation avoids this frequent confusion.

It is also possible to recover μ from a measured period and radius: μ = 4π²r³/T². This relationship estimates a gravitational parameter under the model's assumptions. It does not directly give mass if G is unknown and the other conditions are omitted. Precision also matters: a small relative error in r is amplified by the third power.

5. Kepler's laws and their scope

Kepler's laws describe idealised planetary orbits: an ellipse with the attracting centre at a focus, equal areas swept in equal times, and a relationship between squared period and cubed semi-major axis. The first two were published in 1609 and the third in 1619. This historical reference point, confirmed by NASA's official account, shows a distinction between describing a regularity from observations and subsequently explaining its structure through an interaction model.

For an elliptical orbit, distance from the centre varies. Speed is greater near the closest point and smaller near the farthest point. The area law therefore prevents treating all elliptical orbits as uniform motion. An ellipse has two foci; the attracting centre occupies one of them in the model, not the ellipse's geometric centre.

The third law is $T^2/a^3 = 4\pi^2/[G(M + m)]$ for an ideal Newtonian two-body problem, with a the semi-major axis of their relative orbit. If m is negligible compared with M, GM is often used in the denominator. For a circular orbit, a becomes r. The relationship $T^2 = a^3$ is valid only with suitable units and system; it is not a universal equality between squared seconds and cubic metres.

6. Potential energy suited to large distances

In the central Newtonian model, potential energy is often chosen as zero at infinite distance: $E_p(r) = -GMm/r$. It is negative at finite distance. This negative value is not a defect: the potential energy origin is conventional. Radial force is recovered through $F_r = -dE_p/dr = -GMm/r^2$.

The choice $E_p = mgz$ near the ground is a local approximation of changes in this energy. Do not use mgh with constant g for an

arbitrarily large distance change. Near a surface, the change can be linearised; far from it, the 1/r dependence becomes important.

For a circular orbit, $v^2 = GM/r$ gives $E_c = GMm/(2r)$. Total mechanical energy is therefore $E_m = -GMm/(2r)$. It is negative for this bound orbit. This relationship does not allow a circular radius to be inferred from any energy without examining trajectory shape and velocity. For a bound elliptical orbit, the semi-major axis appears in an analogous formula, but instantaneous distance varies.

7. Escape speed: an ideal energy threshold

In a model without atmosphere and with an isolated attracting centre, the minimum speed allowing infinity to be reached with final speed tending to zero is calculated through $E_m = 0$. Starting at distance r, $mv^2/2 - GMm/r = 0$, hence $v_{lib} = \sqrt{2GM/r}$. It is $\sqrt{2}$ times the circular speed at the same distance.

This expression does not describe a rocket's speed throughout a mission or a launch procedure. Propulsion can act gradually; the atmosphere and other bodies change the constraints. The relationship provides a threshold for an object given an initial impulse and then left without propulsion in a precise model. Replacing it with a “mandatory speed for every rocket” would misinterpret it.

With the fictional data $\mu = 4.00\times10^{14}\ m^3/s^2$ and $r = 8.00\times10^6$ m, ideal escape speed is 1.00×10^4 m/s, whereas circular speed is about 7.07×10^3 m/s. The difference comes from the energy destination: remaining bound on a circle or reaching an arbitrarily large distance. The calculations answer distinct questions.

8. Weightlessness and shared motion

In a freely falling cabin, objects and people have similar gravitational accelerations. The contact force that supported a person on the ground can become very small. This situation is often called weightlessness; it does not demonstrate that the gravitational field has disappeared. Contact and gravitation must be studied separately, as in the lift example in the dynamics chapter.

In an ideal orbital frame, a description can be constructed in which some inertial forces balance gravitation. This does not make those forces additional interactions in the original inertial frame. Using terms from both descriptions simultaneously without specifying the frame causes double counting. At the level treated here, describing centripetal gravitational acceleration remains the most direct approach.

The field is not perfectly uniform across an extended system. Differences in gravitational acceleration between points contribute to tidal effects. A sufficiently small cabin can neglect them for a simple exercise; a more precise problem must account for them. The shared-motion approximation therefore does not claim that every point of every object has exactly the same acceleration in space.

9. A synchronous orbit and additional conditions

An orbit whose period equals a celestial body's rotation period is synchronised with that rotation in the temporal sense. Remaining above the same point on the equator in the simple model additionally requires a circular orbit in the equatorial plane and in the same direction as rotation. Equal periods alone do not guarantee a fixed apparent position.

The relationship $r^3 = \mu T^2/(4\pi^2)$ calculates an ideal synchronous radius when μ and T are given. It does not directly give altitude: the body's radius must then be subtracted. The exercise must distinguish a rotation period from an arbitrarily defined civil day if high precision is required. Choosing 24 hours without examining the supplied data can introduce a conceptual discrepancy.

Even in such a model, a signal takes non-zero time to cross the distance. Propagation and orbital motion are different questions. A satellite appearing fixed is not stationary in a frame centred on the celestial body; it completes a revolution. “Fixed” must always be related to the observer or frame considered.

10. The Newtonian model and more precise models

Newtonian gravitation is very useful for many celestial-mechanics calculations. It is not a sufficient description of every gravitational phenomenon. Precision requirements or particular fields call for other models, notably relativistic ones. Recognising this limit does not make Newtonian exercises useless: a model can be appropriate within a domain without being universal.

The useful opposition is therefore not a simplistic “false old theory versus true new theory”. Examine predictions, conditions and precision. A school problem stated as two-body deliberately neglects other attractions; mission software may include multiple perturbations. The choice depends on the aim and tolerance for discrepancies.

In a report, distinguish given parameters, measured parameters and calculated results. If a period was used to fit μ, subsequently recovering that same period with that μ is not independent verification. Another observation or a prediction not used in fitting is needed. This connects orbital mechanics with the general validation method in the opening chapter.

11. Worked exercises

A - Altitude. A celestial body has radius R; a satellite is at altitude 2R. Compare its field with the surface field. **Answer:** its distance from the centre is 3R, so the ratio is 1/9. Using 2R would give 1/4 and confuse altitude with orbital radius.

B - Period. Two fictional satellites describe circles around the same celestial body, with r_2 = 9r_1. **Answer:** v_2/v_1 = 1/3 and T_2/T_1 = 27. The ratios follow from powers −1/2 and 3/2. Satellite masses are assumed negligible compared with the attracting centre's mass.

C - Satellite mass. Does doubling m while retaining r and M change circular speed? **Answer:** not in the approximation where M is fixed and dominant, because m cancels in the acceleration equation. Force and energies, however, are doubled. Confusing speed with force would conceal this distinction.

D - Units. Check μ's dimension. **Answer:** since $a = \mu/r^2$, μ has dimension $\text{length}^3/\text{time}^2$. In $T^2 = 4\pi^2 r^3/\mu$, the quotient therefore has dimension time^2. Data expressed in km^3/s^2 must be converted before calculation using metres.

E - Interpretation. An object has E_m > 0 in the isolated central model. Can we assert that it follows a circle? **Answer:** no; a bound circular orbit has E_m < 0 when the energy origin is at infinity. Positive energy is compatible with an unbound trajectory, under the model's other conditions. Velocity direction and angular momentum are still needed to specify that trajectory.

12. Revision and sources

Recover circular speed and period from Newton's law and centripetal acceleration. Explain why gravitation remains present in weightlessness, why altitude must be added to the body's radius and why a synchronous period is insufficient to make a satellite appear fixed. Conclude every calculation with a sentence connecting the result to assumptions.

Official reference for Kepler's laws and the dates 1609 and 1619: NASA Science, https://science.nasa.gov/solar-system/orbits-and-keplers-laws/, consulted for the French manuscript on 11 October 2026. This mathematical discussion, comparisons and exercises are LFDP originals; no historical narrative, image or passage from the page is reproduced. The published final-year curriculum, physics component, provides the school framework: https://www.education.gouv.fr/bo/19/Special8/MENE1921249A.htm. Explicitly fictional numerical parameters are not measurements of Earth or an operational trajectory.

Chapter 6 - Fluids: pressure, flow rate and the limits of flow models

Historical reference point - 1648: comparing pressure at different altitudes

Pascal's Récit de la grande expérience de l'équilibre des liqueurs, published in 1648, reports the experiment carried out following his instructions by Florin Périer at the puy de Dôme. The BnF emphasises that the mercury column's height decreased with altitude. This result is a historical example of a comparison capable of distinguishing explanations: the weight of air becomes a cause to examine, rather than invoking nature's abhorrence of a vacuum. This chapter's calculations remain separate from that experiment. They require neither handling mercury nor reproducing an old apparatus.

Reference for the historical fact, consulted for the French manuscript on 11 October 2026: BnF, presentation of the Récit's original edition, https://www.bnf.fr/fr/etienne-klein-resident-musee. LFDP educational analysis, without verbatim quotation or reproduced image.

1. Describing a medium without following every molecule

A fluid can flow and deform under stress. Liquids and gases differ importantly, particularly in compressibility. A macroscopic description uses local quantities such as pressure, density, temperature and velocity. This approach treats the medium as continuous at the relevant scale; it does not claim that matter really lacks microscopic structure.

Density ρ is mass divided by volume, in kg/m^3. It may vary with temperature, pressure or composition. Here, assuming an incompressible fluid means that density changes are negligible in the problem. It does not assert that no real liquid can be

compressed. For a gas, this approximation may sometimes suit certain slow flows, but it must be examined rather than applied by habit.

A steady flow has quantities that do not vary with time at each fixed point. This does not mean that fluid particles are stationary. A particle can pass through regions with different velocities in a steady flow. Separating evolution observed at a point from evolution followed along a particle prepares a correct reading of Bernoulli's equation and streamlines.

2. Pressure and force: specify the surface and direction

Pressure corresponds to normal force per unit area in the hydrostatic model. Its SI unit is the pascal: 1 Pa = 1 N/m². On a small plane surface with uniform pressure, pressure-force magnitude is pA. For an extended surface where pressure varies, add local contributions; pressure measured at one point is not automatically representative of the whole surface.

The pressure force is normal to the surface. Forces on opposite faces of an object may largely balance. High absolute pressure therefore does not necessarily give a large resultant: this often depends on a pressure difference. For a plate separating two media at uniform pressures p_1 and p_2, the normal resultant has magnitude |p_1 − p_2|A, in the direction imposed by the higher pressure.

With a pressure difference of 5 000 Pa and an area of 10 cm², or 0.0010 m², the resultant force is 5.0 N. Using 10 directly would introduce an erroneous factor of ten thousand. Area conversions deserve particular attention: 1 cm² is 10^−4 m², because the length factor is squared. One hectopascal is 100 Pa and one bar 100 000 Pa; these units must not be confused.

3. Statics of a liquid with constant density

Take a stationary liquid with constant density in a uniform gravitational field. With an upward vertical axis z, pressure satisfies dp/dz = −ρg. This can be obtained by balancing a small column: the difference between pressure forces balances weight. Between two levels, p_2 − p_1 = −ρg(z_2 − z_1). Pressure therefore increases downwards.

At depth h below a surface at pressure p_0, pressure is p = p_0 + ρgh. The term ρgh is a pressure difference relative to the surface, not necessarily absolute pressure. For ρ = 1 000 kg/m³, g = 10 m/s² and h = 0.50 m in an exercise, this difference is 5 000 Pa. If p_0 = 100 000 Pa, absolute pressure is 105 000 Pa.

In a connected, homogeneous liquid at rest, two points at the same height have the same pressure under these assumptions. Container shape does not directly enter the hydrostatic difference. A narrow column and a broad tank can therefore produce the same pressure at the same depth. This does not mean they contain the same mass or that forces on all their walls are identical.

The result does not apply unchanged to density varying strongly with altitude, an accelerated fluid or a flow. In the atmosphere, for example, ρ is generally not constant across a large altitude difference. A simple law can be appropriate over a small height and become insufficient when the scale changes.

4. Archimedes' principle and displaced volume

Pressure is higher below an immersed object than above it. Adding pressure forces gives an upward resultant equal to the weight of displaced fluid in the hydrostatic model. Its magnitude is ρ_f gV_immergé for a homogeneous fluid, with ρ_f the fluid's density. The relevant volume is the immersed volume, not necessarily the object's total volume.

For a fully immersed prismatic block, the formula can be understood without following a curved surface. Lateral forces balance. If height is h and horizontal area A, the pressure difference between bottom and top is ρ_f gh. The resultant is therefore ρ_f ghA = ρ_f gV. This demonstration explains why detailed shape is not the principal factor in the general relationship.

A fictional object of mass 0.60 kg and total volume 0.0010 m³ is fully immersed in a liquid of density 1 000 kg/m³, with g = 10 m/s². Its weight is 6.0 N and buoyancy 10 N. If no other force restrains it, the initial vertical resultant is upward. To float in equilibrium, it must displace 0.60 kg of liquid, or 0.00060 m³: 60 % of its volume is immersed. This does not necessarily mean 60 % of its height if its cross-section varies.

Buoyancy depends on average density and the volume available to displace fluid, not solely on the material of a wall. A hollow object can therefore float even if its material is denser than the liquid. This reasoning certifies no real load capacity: stability, watertightness, fluid motion and mechanical strength are other questions.

5. Volume flow rate and conservation of mass

Volume flow rate Q is volume crossing a section per unit time, in m³/s. If uniform velocity normal to area A is appropriate, Q = Av. If velocity varies across the section, an appropriate average velocity or integral can be used. The simple formula therefore does not mean that every particle travels at the same speed in a real pipe.

For steady flow without accumulation or leakage, incoming mass per unit time equals outgoing mass. Mass flow rate is ρQ in a uniform section. If density is constant, Q_1 = Q_2 and A_1v_1 =

A_2v_2. For a compressible fluid, retain ρ_1A_1v_1 = ρ_2A_2v_2; removing densities without justification changes the balance.

A fictional flow rate of 0.0020 m³/s crosses an area of 0.0040 m²: average speed is 0.50 m/s. In a section four times smaller, 0.0010 m², it is 2.0 m/s if flow rate and density are conserved. Flow rate is 2.0 litres per second, since 1 litre is 10^−3 m³. Narrowing here increases average speed, not the amount of liquid passing each second.

At a branch, incoming flow rate equals the sum of outgoing flow rates if the same incompressible, steady model applies. Equal pressure in branches does not guarantee identical flow rates: lengths, diameters and pressure losses may differ. Conservation sets an overall constraint; it does not supply all the relationships needed to determine the distribution.

6. Bernoulli: conditional conservation

For steady, incompressible flow without significant viscous dissipation, one form of Bernoulli's equation is p + ρgz + ρv²/2 = constant along a streamline, with no work added or removed between the points considered. All three terms have the dimension of pressure. This relationship is not a universal rule saying that any high speed everywhere implies low pressure.

At equal heights, it gives p_2 − p_1 = (ρ/2)(v_1² − v_2²). For speeds of 0.50 and 2.0 m/s from the previous example, with ρ = 1 000 kg/m³, the difference is 500(0.25 − 4) = −1 875 Pa. Static pressure at the second point is lower in this model. The relationship has a stated domain: it does not determine losses in a long real pipe on its own.

The term ρv²/2 is sometimes called dynamic pressure. It must not be confused with static pressure measured by any arbitrary device. An instrument, its orientation and how it disturbs flow determine

what it measures. Mathematical expression and experimental observation must be connected through a procedure, as for the guide's other models.

The assumptions are confirmed by NASA Glenn's official presentation of Bernoulli's equation, consulted for the French manuscript on 11 October 2026: https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/bernoullis-equation/. The discussion and numbers here are original; no diagram or demonstration from that page is reproduced.

7. A reservoir and ideal outlet speed

Consider a reservoir whose free surface is very large compared with a small outlet. In an ideal model, surface velocity can be neglected, pressures at the surface and outlet are identical, and flow is treated as approximately steady. Bernoulli gives v_sortie = sqrt(2gh), where h is the level difference between surface and outlet. This relationship is often called Torricelli's law.

With h = 1.25 m and g = 10 m/s² for calculation, ideal speed is 5.0 m/s. If the free surface falls, h changes, and so does ideal speed. This value therefore cannot be retained throughout emptying without discussing the changing level. Flow rate also depends on the effective cross-section and departures from the ideal model, notably flow contraction and losses.

The similarity to the speed acquired in free fall through the same height reveals a shared energy structure. It does not imply that the whole flow is identical to a particle's motion. Pressure, geometry, continuity and wall contact bring conditions particular to the fluid. An analogy is useful when we specify what it compares and what it leaves out.

8. Viscosity and pressure losses

Viscosity expresses resistance associated with deformations and velocity differences between fluid layers. It can dissipate mechanical energy. In a real pipe, maintaining a flow rate may require a pressure difference even between sections of equal height and equal average speed. Ideal lossless Bernoulli flow is insufficient to describe this situation.

A pressure-loss term can be introduced into an energy balance, provided its definition and unit are specified. A dissipative pressure drop is not a disappearance of mass or total energy; part of mechanical energy becomes internal energy. Pumps can add energy, and other devices remove it. System boundaries become essential again.

As an extension of scientific knowledge, consider steady, fully developed laminar flow of an incompressible Newtonian fluid with constant viscosity in a straight horizontal cylindrical tube of constant radius, with no slip at its fixed walls. Its volume flow rate can be described by $Q = \pi r^4 \Delta p/(8\eta L)$, where η is dynamic viscosity, L the length of the section studied, r its internal radius and Δp upstream pressure minus downstream pressure; Q is positive in this direction. This law is given as a particular model, not a relationship applicable to every pipe. It shows strong dependence on radius and a difference from ideal conservation without viscosity. Turbulence, entrances, bends or changing properties require other descriptions.

9. Transmitting force does not create energy

In an ideal stationary hydraulic device, a pressure change can be transmitted through a liquid and produce different forces on pistons with different areas. If the same pressure difference acts on A_1 and A_2, then F_2/F_1 = A_2/A_1. A large area can therefore receive a greater force. This does not multiply work for free.

Conservation of displaced volume imposes A_1d_1 = A_2d_2 in the incompressible model without leaks. Greater force corresponds to smaller displacement, and F_1d_1 = F_2d_2 in the ideal case. For an area ratio of 10, a force of 10 N on the small piston corresponds to 100 N on the large one; displacement of 5 cm for the small one corresponds to 0.5 cm for the large one. Both amounts of work are 0.50 J.

Friction and deformation reduce real useful work. These calculations are conservation exercises, not manufacturing or lifting instructions. Maximum pressure, material strength, stability and safeguards are not studied. A physics formula is insufficient to validate an apparatus.

10. Worked exercises

A - Relative pressure. A liquid of density 800 kg/m³ has a surface at 100 000 Pa. With g = 10 m/s², calculate pressure at a depth of 2 m. **Answer:** the hydrostatic difference is 16 000 Pa; absolute pressure is 116 000 Pa. Answering 16 000 Pa without stating that it is relative pressure would be ambiguous.

B - Cross-section. A flow rate of 3 litres per second crosses an area of 6 cm². **Answer:** Q = 0.003 m³/s and A = 0.0006 m², hence v_moy = 5 m/s. If area doubles at conserved flow rate, speed halves. This result uses constant density and describes no guaranteed real device.

C - Floating. A homogeneous object of density 600 kg/m³ floats in a liquid of density 1 000 kg/m³. **Answer:** in ideal equilibrium, ρ_obj gV_total = ρ_liquide gV_immergé. The immersed volume fraction is 0.60. The same height fraction requires a shape with an appropriately constant cross-section.

D - Bernoulli. In ideal horizontal flow, speed is the same at points 1 and 2, but measurement shows falling pressure. **Answer:** this observation calls for examining assumptions: dissipation, energy added or removed, steadiness and measurements. Do not force the real result into the ideal relationship by deleting a datum. The model may be insufficient or the procedure flawed.

11. Revision and scope

For revision, connect each formula with its balance: forces for statics, volume or mass for flow rate, energy for Bernoulli. Always check whether pressure is absolute or relative, velocity local or averaged, and volume total or immersed. These distinctions often matter more than computational difficulty.

School framework: published final-year curriculum, physics component, https://www.education.gouv.fr/bo/19/Special8/MENE1921249A.htm. Additional official reference: NASA Glenn on Bernoulli, linked above. Original LFDP writing and exercises, without verbatim quotation or reproduced figure. Ideal cases authorise no dangerous pressure manipulation, diving, household installation or care device.

Chapter 7 - Electricity: charge, energy and the evolution of an RC circuit

Historical reference point - 1800: correspondence about electrical phenomena

The Royal Society holds a letter from Alessandro Volta to Joseph Banks dated 20 March 1800, concerning electricity associated with contact between conducting substances of different kinds. This archival source places electrical phenomena within research based on devices and scientific exchange. Uncritically equating this historical object with the exercises' ideal source would be misleading. The chapter precisely separates the real phenomenon from the relationships chosen to model it: voltage, current, resistance and capacitive storage answer different questions.

Reference for the historical fact, consulted for the French manuscript on 11 October 2026: Royal Society, archives L&P/11/137, https://makingscience.royalsociety.org/items/l-and-p_11_137/letter-account-of-electricity-excited-by-contact-and-conducting-substances-of-different-kinds-from-alessandro-volta-to-joseph-banks. LFDP educational analysis, without verbatim quotation or reproduced image.

1. Electric charge and current

Electric charge is a signed quantity expressed in coulombs. A system can carry positive charge, negative charge or zero net charge. Current corresponds to charge transfer per unit time. For an amount of charge Δq crossing a section during Δt, average current is $\Delta q/\Delta t$; instantaneous current is the derivative of transferred charge along a chosen orientation. One ampere is one coulomb per second.

Conventional current points in the direction of positive-charge movement. In a metal, mobile electrons move opposite to conventional current. In other media, several types of carrier may

contribute. The sign of current in an equation therefore depends on an orientation and must not be equated solely with one particular electron's direction of movement.

A constant current of 2.0 mA for 3.0 s corresponds to transferred charge of 6.0 mC. If current varies, integrate i(t) over the duration or calculate a signed area under its graph. A final current value does not necessarily describe the whole phase. This distinction resembles that between final velocity and displacement in accelerated motion.

2. Voltage and potential difference

A voltage U_AB is a potential difference V_A − V_B, expressed in volts. One volt is one joule per coulomb. Voltage is not an amount of stored charge, and current is not moving voltage. The quantities describe different aspects: potential between two points and charge flow in a branch.

Reversing the points changes the sign: U_BA = −U_AB. In an ideal lumped-element circuit where omitted induction effects are negligible, potential differences add along a path. Their signed sum around a closed loop is zero. This loop rule does not mean adding only the positive values stated for components; orientations matter.

Charge conservation gives the junction rule in the model without significant charge accumulation at the junction: sum of incoming currents = sum of outgoing currents. If one incoming current is 5 mA and two outgoing currents are 2 and 3 mA, the balance is consistent. A charge-storing component such as a capacitor remains compatible with this conservation: storage must be included in the system and its terminals.

3. Resistance and the power convention

For an ideal ohmic conductor within a given operating range, u = Ri, with R in ohms. Adopt the passive sign convention: current enters the terminal whose potential is taken as positive for u. Electrical power received is then p = ui. For resistance R > 0, p = Ri² = u²/R is positive and corresponds to a transfer into internal energy in the model.

A resistance of 1.0 kΩ subject to 5.0 V carries current of 5.0 mA and receives power of 25 mW. If these values remain constant for 20 s, energy received is 0.50 J. Convert prefixes before applying the laws. Confusing kΩ with Ω would multiply current by a thousand; confusing mW with W would multiply energy by a thousand.

The ohmic relationship is not universal for every component. A diode, a lamp whose temperature changes substantially or a real source may require other models. A resistance stated in an exercise is a model parameter, not a guarantee of behaviour under every voltage. No calculation in this section validates an installation, power supply or real assembly.

4. Simple combinations and model boundaries

For resistors in series carrying the same current, voltages add and R_eq = R_1 + R_2. For resistors in parallel subject to the same voltage, currents add and 1/R_eq = 1/R_1 + 1/R_2. These equalities follow from loop and junction rules; do not apply them to two elements merely because they appear close in a diagram.

Two identical resistors of 2 kΩ give 4 kΩ in series and 1 kΩ in parallel. The equivalent resistance of positive parallel branches is less than the smallest individual resistance. This provides an order-of-magnitude check. In series, it is instead greater than each. A reversed answer often signals confusion about topology.

A real source can be described within a limited range by electromotive force E and internal resistance r: voltage supplied to a load becomes U = E − ri if current leaves the source. This voltage drop reminds us that ideal components have limits. The model must not be used to estimate a short circuit or dangerous situation without data; protections, transient regimes and real characteristics are absent.

5. An ideal capacitor and capacitance

An ideal capacitor has two plates carrying opposite charges in the simple model. If q denotes the charge on plate A and u = V_A − V_B, write q = Cu with constant C, in farads. One farad is one coulomb per volt. The sign of q is related to that of u; positive capacitance does not mean that charge is always positive.

Under the passive sign convention with current entering plate A, i = dq/dt = Cdu/dt. The capacitor can thus receive current when voltage varies, although no charge crosses the ideal dielectric between plates. Saying it “blocks current” without specifying the regime is misleading: under steady constant voltage, ideal current is zero; during a transient, it can be non-zero.

A capacitance of 100 µF subject to 5.0 V corresponds to charge of 500 µC, or 0.00050 C, on the positive plate. If voltage rises linearly from 0 to 5 V in 0.50 s, this model's current is C×10 V/s = 1.0 mA. A different rise shape would produce another current. Capacitance links charge and voltage; it does not determine an evolution's duration on its own.

6. Stored energy and capacitor combinations

When an infinitesimal charge dq is transferred into the capacitor, energy received is u dq. With u = q/C, integration from zero to q gives E_capa = $q^2/(2C) = Cu^2/2$. The factor 1/2 arises because voltage increases during transfer; simply multiplying final voltage by all the charge would double stored energy.

For C = 100 µF and u = 5.0 V, energy is 0.00125 J, or 1.25 mJ. This energy is distinct from the charge of 500 µC. Two capacitors with different capacitances can have the same voltage and different energies. A comparison must therefore specify which quantity is shared.

In parallel, voltages are identical and charges add: C_eq = C_1 + C_2. In series, for the model of two initially neutral capacitors with an isolated intermediate junction, charge magnitudes are identical and 1/C_eq = 1/C_1 + 1/C_2. For 2 µF and 3 µF, equivalent capacitances are 5 µF in parallel and 1.2 µF in series.

Under a total of 10 V in series, charge is 12 µC. Voltages are then 6 V across the capacitor of 2 µF and 4 V across the one of 3 µF. Their sum recovers 10 V. Equal voltage distribution is not automatic; here it requires equal capacitances. Initial conditions and pre-existing charges can also alter a more general problem.

7. Charging an RC circuit: establishing the equation

An ideal constant-voltage source E supplies a resistor R and capacitor C in series. Choose i as charging current entering the capacitor's positive plate. The loop rule gives E = Ri + u_c, and i = Cdu_c/dt. We obtain RCdu_c/dt + u_c = E. This equation describes the transient under the stated ideal assumptions.

Setting τ = RC, the solution with u_c(0) = u_0 is u_c(t) = E + (u_0 − E)e^(−t/τ). Check it by differentiation and substitution. At t = 0,

the expression gives u_0; in the long term, it tends to E. A zero initial condition gives u_c = E[1 − e^(−t/τ)] and i = (E/R)e^(−t/τ).

Current is initially E/R in this model, then decreases. Capacitor voltage changes continuously when current is finite, unlike the ideal voltage imposed by the source when the phase changes. Switching therefore does not permit arbitrary resetting of u_c to zero. If a previous charge remains, it becomes the new initial condition.

8. Time constant and threshold precision

The product RC has the dimension of time: Ω×F = s. With R = 10 kΩ and C = 100 µF, τ = 1.0 s. For E = 5.0 V and u_0 = 0, after one time constant, voltage is 5(1 − e^−1) ≈ 3.16 V, about 63.2 % of its final value. At three time constants it reaches about 95.0 %, and at five about 99.3 %.

The final value is never reached exactly in finite time in this ideal solution. “Charging complete” therefore means that a precision criterion has been chosen. To reach at least 99 % of E, solve 1 − e^(−t/τ) ≥ 0.99, giving t ≥ −τ ln(0.01) ≈ 4.605τ. This threshold is more precise than the absolute assertion “after five τ, it is exactly finished”.

A graph can also estimate τ. At the origin, the tangent to the charging curve of an initially neutral capacitor has gradient E/τ and meets the level E at t = τ. This construction assumes a properly calibrated scale and appropriate model. A graphical estimate has uncertainty; it does not equal an exact parameter simply because a line was drawn.

9. Discharging and changing convention

A capacitor initially at U_0 is then connected to a resistor without an ideal source. Its voltage satisfies RCdu_c/dt + u_c = 0, hence u_c(t) = U_0e^(−t/τ). If i_d is defined as current leaving the positive plate through the resistor, i_d = u_c/R = −Cdu_c/dt. The minus sign comes from the decreasing charge on this plate.

If current were kept defined as entering the plate, it would be negative during this discharge for U_0 > 0. Both descriptions are consistent; changing convention without saying so creates only an apparent contradiction. The diagram or sentence defining current must therefore accompany the equation.

Capacitor energy varies as e^(−2t/τ), because it is proportional to u_c². It does not decrease according to the same exponential as voltage. At t = τ, voltage is about 36.8 % of its initial value, but energy about 13.5 %. This difference is a good exercise in interpreting quantities: a time law does not transfer unchanged to all related quantities.

10. An energy balance for charging

For ideal charging from zero to E through R, the source transfers total charge CE. Its supplied energy is therefore CE², because its voltage remains E. The capacitor stores CE²/2; the other half is dissipated in the resistor in this model. This may be surprising, because increasing or decreasing R changes duration and current but not this total distribution in the ideal model.

Do not extend this result to R = 0 without examination. The model then predicts a transient of zero duration and unbounded current, signalling insufficient physical assumptions. Real resistances, inductances, source limits and radiation effects become relevant. A mathematical limit can reveal a model's domain rather than describe a possible device.

Comparison with a battery again recalls the distinction between energy and capacity. Battery capacity expressed in Ah is an amount of charge; an estimate in Wh also uses voltage. For example, 2 Ah at nominal voltage of 3.7 V corresponds to 7.4 Wh in a nominal estimate. Because voltage changes during use, this multiplication guarantees neither the energy actually available nor a device's operating time.

11. Worked exercises

A - Units. R = 2.0 kΩ and C = 250 µF. **Answer:** τ = 2 000×250×10^−6 = 0.50 s. Multiplying the values 2 and 250 without prefixes does not give time in seconds.

B - Initial condition. A charging circuit has E = 6 V, τ = 2 s and u_c(0) = 2 V. **Answer:** u_c(t) = 6 − 4e^(−t/2). At t = 0, voltage is 2; in the long term, 6. Using 6[1 − e^(−t/2)] would forget the initial charge.

C - Discharge threshold. When does u_c become at most one tenth of U_0? **Answer:** e^(−t/τ) ≤ 0.10, hence t ≥ τ ln(10) ≈ 2.303τ. This calculation is a theoretical relative criterion, not an electrical-safety checking instruction.

D - Relative energy. When voltage has been divided by three, what happens to energy? **Answer:** it is divided by nine, because E_capa = Cu²/2 for constant C. Division by three would correspond to a linear relationship, which is not the energy relationship.

E - Steady-state model. An ideal capacitor has long been subject to constant voltage. **Answer:** du_c/dt = 0, so ideal current is zero. Its charge and energy may remain non-zero. Absence of current does not mean absence of every stored electrical quantity.

Extension - Capacitance geometry and the principle of a sensor

For two parallel plane plates separated by a homogeneous insulator, the model without edge effects gives C = εS/d. Area S is that of the facing surfaces, d their separation and ε the medium's permittivity, expressed in F/m. In a vacuum, ε = ε0; for an ideal linear dielectric, write ε = ε0εr. This relationship notably assumes that lateral dimensions are large compared with d. It does not exactly describe an arbitrarily shaped component or a medium with strongly varying properties.

With material and separation fixed, doubling S doubles C. With material and area fixed, doubling d halves C. A deformation changing d can therefore be detected through a capacitance change: this is the principle of a capacitive sensor model. Capacitance is not directly pressure, position or humidity; a conversion relationship and calibration appropriate to the device are needed. Paper calculations do not prove a real sensor's accuracy.

Original exercise. In a fictional model treating air as a vacuum, S = 20 cm², d = 1.0 mm and ε0 = 8.85 × 10^{-12} F/m. Calculate C, then its change if d increases by 10 %, with other parameters constant. Finally, determine the effect on the time constant if this capacitor is combined with R = 10 MΩ in the ideal RC circuit studied.

Answer. Conversion gives S = 20 × 10^{-4} m² = 2.0 × 10^{-3} m² and d = 1.0 × 10^{-3} m. Thus C = 8.85 × 10^{-12} × 2.0 = 1.77 × 10^{-11} F, or 17.7 pF. Increasing d by 10 % gives C' = C/1.10, about 16.1 pF: the relative decrease is 9.09 %, not exactly 10 %. Calculate the reciprocal ratio before rounding. The initial time constant is RC = 1.77 × 10^{-4} s, or 177 µs; it is also divided by 1.10 when capacitance changes.

Only for a small change, $\Delta C/C \approx -\Delta d/d$ at constant area and permittivity. This first-order approximation is useful for interpreting sensitivity, but is not the exact equality used in the exercise. If ε changes along with d, one measurement of C can correspond to several causes. Distinguishing a detected variation from an identified cause prevents assigning a unique property to a signal too quickly.

References for the relationship and principle, consulted for the French manuscript on 11 October 2026: University of Lille, theory of parallel-plate capacitors, https://tpmpathome.univ-lille.fr/capacites_condensateurs/version_papier/_papier.pdf ; official Éduscol question presenting a capacitive sensor model, https://eduscol.education.gouv.fr/sites/default/files/document/22-pycj1nc1pdf-97374.pdf. This exercise and its numbers are original; no high-voltage manipulation or reproduction of an examination question is proposed.

12. Revision and references

Reconstruct an RC differential equation from the loop rule and $i = Cdu/dt$. Check its solution by substitution, then explain initial conditions, the limit and a chosen threshold. Revise units and sign conventions before memorising expressions. The same reasoning subsequently serves other systems with exponential evolution.

School framework: published final-year curriculum, physics component, https://www.education.gouv.fr/bo/19/Special8/MENE1921249A.htm and appendix https://cache.media.education.gouv.fr/file/SPE8_MENJ_25_7_2019/92/9/spe249_annexe_1158929.pdf. Units: BIPM, https://www.bipm.org/en/publications/si-brochure/, consulted for the French manuscript on 11 October 2026. The chapter, balances and exercises are LFDP originals. These examples teach no mains-powered assembly, battery manipulation or electrical-safety procedure; theory does not validate an installation.

Chapter 8 - Thermodynamics: distinguishing a state from the transfers changing it

Historical reference point - 1859: comparing radiation absorption by gases

At the Royal Institution, John Tyndall began assembling an apparatus in 1859 to study absorption and transmission of thermal radiation by different gases. The institution holds a tube from this equipment and connects the research with water vapour's role in the natural greenhouse effect. This reference point does not attribute every stage of climate science to one scientist. It illustrates an approach relevant to the chapter: distinguish the media's radiative properties before completing an energy balance. A gas must not be assumed equally transparent at every wavelength.

Reference for the historical fact, consulted for the French manuscript on 11 October 2026: Royal Institution, account of Tyndall's apparatus, https://www.rigb.org/explore-science/explore/collection/john-tyndalls-radiant-heat-apparatus. LFDP educational analysis, without verbatim quotation or reproduced image.

1. Choosing the system and exchanges

Thermodynamics describes systems containing very many particles through macroscopic quantities. Temperature, pressure, volume and amount of substance describe a state without following every particle. This description does not replace microscopic structure; it provides another modelling level suited to balances and transformations.

A closed system exchanges no matter with its surroundings but can exchange energy. An open system exchanges matter; the

corresponding flows require a more complete balance. An isolated system exchanges neither matter nor energy in the model. An adiabatic system exchanges no energy by heat transfer but can receive or supply work. These terms are not synonyms: adiabatic compression can change temperature.

Before any equation, draw the boundary and name what can cross it. A liquid alone heated by a resistor receives a transfer from that resistor. A combined liquid-and-resistor system can receive energy as electrical work. The overall result must remain consistent, but balance terms depend on the chosen boundary. A conservation formula without a defined system remains ambiguous.

2. Temperature and internal energy

Thermodynamic temperature is expressed in kelvins. Its relationship with degrees Celsius is $T(K) = \theta(°C) + 273.15$. A temperature difference has the same numerical value in kelvins and degrees Celsius, but a ratio of thermodynamic temperatures requires kelvins. Saying 40 °C is “twice as hot” as 20 °C therefore has no meaning as a thermodynamic ratio.

Internal energy U describes microscopic contributions, notably constituent motion and interactions. It differs from the kinetic energy of overall motion and potential energy associated with overall position. A stationary object can have substantial internal energy. A macroscopic velocity does not measure all molecular agitation.

Temperature is not an amount of energy. Two systems at the same temperature can have different masses, heat capacities and internal energies. Heat transfer is not a stored substance called “heat” either: it is a way of transferring energy. In writing, say that a system receives energy by heat transfer and its internal energy changes, rather than that it “contains a transfer”.

3. The ideal gas: a conditional equation of state

The ideal-gas model relates absolute pressure p, volume V, amount of substance n and temperature T through pV = nRT. The constant R can be taken as 8.314 J/(mol·K) for exercises. Volume must be in m³ if pressure is in pascals. Pressure relative to the atmosphere and Celsius temperature cannot be inserted directly into this relationship.

For n = 0.10 mol, T = 300 K and V = 0.0020 m³, calculated pressure is 0.10×8.314×300/0.0020 ≈ 1.25×10^5 Pa. Unit consistency follows from Pa·m³ = J. If V doubles with n and T constant, pressure halves. If T doubles with n and V constant, pressure doubles, but this is doubling in kelvins.

The model notably neglects some interaction effects and particles' own volume. It becomes less suitable when these effects are important, for example near condensation. Do not assume that a real gas obeys this law exactly at every pressure and temperature. Additional official reference: NASA Glenn, equation of state, https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/equation-of-state/, consulted for the French manuscript on 11 October 2026.

4. The first law and sign convention

For a closed system, a general balance can be written ΔU + ΔE_c + ΔE_p = W + Q, where W and Q are energies received respectively through work and heat transfer. If overall mechanical changes are negligible, write ΔU = W + Q. Energy received is positive in this convention; energy supplied to the surroundings is negative. Other conventions exist, but they must be stated and maintained.

Work and heat transfer depend on the transformation path, whereas internal energy change depends on initial and final states. Two transformations between the same states can involve different work and heat transfers whose sum gives the same ΔU. This

distinction between a state quantity and a transfer lies at the balance's heart; it is not a vocabulary subtlety.

For expansion against constant external pressure, work received by the system is W = −p_ext(V_f − V_i), within the stated mechanical model. If volume changes from 2.0 to 4.0 litres under 100 000 Pa, W = −200 J. If the system simultaneously receives Q = 500 J, its internal energy change is +300 J. Negative work means that the system supplies mechanical energy to its surroundings during expansion.

5. Heat capacity and temperature change

For an incompressible system of mass m whose specific heat capacity c is assumed constant over the range studied, an internal energy change without phase change can be written ΔU = mcΔT. Total heat capacity C = mc is expressed in J/K; c in J/(kg·K). The symbol C here differs from electrical capacitance in the previous chapter, despite being the same letter. The unit immediately identifies the quantity used.

If the only relevant exchange is heat transfer, Q = mcΔT. If electrical or mechanical work is also received, include it. A fictional object of mass 0.20 kg, with c = 1 000 J/(kg·K), receives 4 000 J without losses or phase change. Its temperature increases by 4 000/(0.20×1 000) = 20 K, an increase of 20 °C. Twice the mass would require twice the energy for the same increase in this model.

Heat capacity can vary with temperature. The constant-c formula is then an approximation; a more precise calculation can integrate a capacity depending on T. The container, losses and exchanges with air also matter. An ideal estimate is not a guaranteed heating time for real equipment.

6. Mixing and phase change

Two masses of the same substance without phase change, with the same constant specific heat capacity, can be studied through an isolated balance. If the container and losses are neglected, m_1c(T_f − T_1) + m_2c(T_f − T_2) = 0. Final temperature is therefore a mass-weighted mean. It lies between the initial temperatures for positive masses.

In an exercise, 0.20 kg at 60 °C and 0.30 kg at 20 °C give T_f = (0.20×60 + 0.30×20)/0.50 = 36 °C. The simple mean of 40 °C would be incorrect because the masses differ. Celsius can be used for this calculation of differences and a weighted mean, because the common offset of 273.15 is retained; this does not justify using Celsius in temperature ratios or pV = nRT.

During a phase change at a given pressure, energy can be received without temperature changing while phases coexist. A relationship Q = mL uses a specific latent heat L. For a fictional substance with L = 2.0×10^5 J/kg, transforming 0.10 kg requires 20 000 J in the model. The formula mcΔT does not describe this phase, because internal energy changes in a way other than a simple temperature change.

7. Three modes of heat transfer

Conduction transfers energy through a medium without necessarily requiring overall macroscopic movement of matter. Convection associates exchanges with fluid motion. Radiation transfers energy through electromagnetic waves and can occur through a vacuum. Several modes coexist in real situations; separating them in a model allows their contributions to be examined.

For a homogeneous plane wall of area A, thickness e and thermal conductivity λ, a steady one-dimensional model gives heat-transfer power magnitude P = λA|ΔT|/e. Thermal resistance is defined as

R_th = e/(λA), in K/W. Thus P = |ΔT|/R_th. Energy passes spontaneously from the hotter side to the colder one in this case, without another imposed mechanism.

With e = 0.10 m, λ = 0.050 W/(m·K), A = 2.0 m², R_th = 1.0 K/W. For a difference of 20 K, power is 20 W. Doubling thickness halves this power in the model. Contacts, thermal bridges, temperature variations and surface exchanges can alter a real wall. A simple formula cannot establish a building's full performance.

8. Radiation and absolute temperatures

A grey-body model can give radiated power P = εσAT^4, where ε is emissivity between zero and one in this model, σ the Stefan-Boltzmann constant and T absolute temperature. For exchange with an ideal environment at T_env, a simplified net power is εσA(T^4 − T_env^4), with appropriate geometric and radiative assumptions. The fourth power requires kelvins, never degrees Celsius.

With ε = 0.80, A = 0.10 m², T = 400 K, T_env = 300 K and σ ≈ 5.67×10^−8 W/(m²·K^4), illustrative net power is about 79.4 W. This calculation does not mean that every real surface at this temperature has the same emissivity or exchanges. The medium, other surfaces and spectrum may require a more detailed description.

Physical constant values can be checked with NIST: https://physics.nist.gov/cuu/Constants/Table/allascii.txt, table consulted for the French manuscript on 11 October 2026. The rounded values above serve the exercises. The radiative model recalls why a small relative temperature increase can appreciably change power, without itself predicting temperature in a complete system.

9. Cooling in contact with a thermal reservoir

A phenomenological Newton model can write power received as P = −hA(T − T_ext), with hA assumed constant and an environment at constant temperature T_ext. For a system of total heat capacity C, C dT/dt = −hA(T − T_ext). The solution is T(t) = T_ext + (T_0 − T_ext)e^(−t/τ), with τ = C/(hA). Temperature difference, not absolute temperature itself, decreases exponentially.

For a fictional object initially at 80 °C in an environment at 20 °C, with τ = 10 min, temperature after one time constant is 20 + 60/e ≈ 42.1 °C. To reach at most 30 °C, we need 60e^(−t/τ) ≤ 10, hence t ≥ τ ln(6) ≈ 17.9 min. These figures describe an ideal curve, not instructions for handling or touching a hot object.

The equation resembles that of an RC circuit. The comparison concerns mathematical structure: a difference from a final state decreases with a characteristic time. It does not make the mechanisms identical. Cooling can involve radiation, evaporation or changing h, and the thermal reservoir may not remain constant. An exponential fit must be discussed in light of observations and assumptions.

10. Conservation and the direction of transformations

The first law imposes an energy balance but does not alone determine which transformations occur spontaneously. Two isolated objects do not spontaneously exchange thermal energy so that the hotter becomes still hotter and the colder still colder in the usual case studied. The direction of evolution requires additional information, developed through the second law and entropy. This extension is mentioned to avoid asking the first law for a conclusion it does not supply.

A heat pump can transfer energy from a colder medium to a warmer one by receiving work. If it supplies 600 J to the warm side

and receives 200 J of work, it can extract 400 J from the cold side in an ideal balance. The ratio 600/200 = 3 is not an efficiency for creating energy: it compares useful heat supplied with work paid for, while the other input comes from the cold medium.

The coefficient of performance can thus exceed one without contradicting conservation. It must be distinguished from efficiency defined as useful energy divided by all energy received. Temperature conditions and real characteristics impose further limits. This fictional example establishes no appliance performance.

11. A simplified radiative balance and its limits

For a sphere receiving solar flux S, the area intercepting radiation is πR², whereas its total area is 4πR². With albedo a, ideal absorbed power is (1 − a)SπR². If it radiates as a uniform black body, equilibrium gives 4πR²σT^4 = (1 − a)SπR², hence T = [(1 − a)S/(4σ)]^(1/4).

With illustrative parameters S = 1 360 W/m² and a = 0.30, we obtain about 255 K. Radius cancels. This temperature corresponds to the simplified radiative model; it is not automatically the actual average surface temperature. Atmosphere, wavelength-dependent absorption and emission, circulation, day-night alternation and temperature distribution are not described. A mean of T^4 is not exactly the fourth power of a mean of T.

This calculation helps us understand a balance's structure and albedo's role. It is not a climate forecast. A richer model must specify the exchanges and layers included. A simplified value can be informative if we state what it measures; it becomes misleading if presented as a direct measurement of the whole Earth-atmosphere system.

Extension - The greenhouse effect and the radiative balance boundary

Earth's warmed surface emits predominantly infrared radiation. Some atmospheric gases, notably water vapour and carbon dioxide, absorb and emit in bands within this range. Part of the atmosphere's emitted radiation travels towards the ground. This transfer contributes to a surface temperature higher than in a model where radiation would leave directly for space. The atmosphere is therefore not a wall preventing all energy from escaping: it participates in radiative exchanges and other transfers.

The top-of-atmosphere balance compares solar energy absorbed by the entire Earth-atmosphere system with its emission into space. The surface balance also includes atmospheric radiation received and exchanges through convection and evaporation. Mixing these boundaries counts an internal transfer as new energy from outside. Absorption followed by emission does not create energy. These distinctions explain why the previous model's 255 K is not automatically average surface temperature.

A paper-based extension: assume a black surface and an ideal atmospheric layer transparent to solar radiation but fully absorbing surface infrared radiation. Neglect convection and evaporation; let F denote the mean solar flux actually absorbed by the surface. The layer emits σT_a^4 upwards and the same downwards. Its balance requires σT_s^4 = 2σT_a^4. The surface balance gives F + σT_a^4 = σT_s^4. We infer σT_a^4 = F and σT_s^4 = 2F. Outgoing flux to space remains F, equal to incoming flux; the additional downward transfer is internal to the system.

Original exercise. For F = 240 W/m² and σ = 5.67 × 10^{-8} W·m^{-2}·K^{-4}, calculate the model's two temperatures. What happens to the balance if outgoing flux to space is incorrectly replaced by

σT_s^4? Answer: T_a = (F/σ)^(1/4) ≈ 255 K and T_s = (2F/σ)^(1/4) ≈ 303 K. Their ratio is 2^(1/4), about 1.189. Using σT_s^4 as outgoing flux would give 480 W/m² and overlook the layer absorbing that radiation before emitting into space. The apparent difference demonstrates not a violation of conservation but an ill-defined boundary.

This single-layer model is deliberately very simplified: the real atmosphere absorbs differently at different wavelengths, has a temperature profile and also exchanges energy through motion and phase changes. Calculated values are neither measurements of the current climate nor projections. The approach demonstrates a mechanism and conservation; alone, it does not quantify the effect of an actual gas concentration or future scenario.

Qualitative and boundary references, consulted for the French manuscript on 11 October 2026: NASA Science, radiation budget, https://science.nasa.gov/ems/13_radiationbudget/ ; Earth-atmosphere energy balance, https://science.nasa.gov/earth/earth-observatory/climate-and-earths-energy-budget/. The single-layer grey model, its demonstration and the exercise above are original LFDP educational developments; no NASA figure or climate series is reproduced.

12. Worked exercises and revision

A - Work's sign. A gas is compressed from 3 to 2 litres under constant external pressure of 100 000 Pa. **Answer:** W = −p_extΔV = +100 J. If Q = −40 J, ΔU = +60 J. Compression supplies work to the system despite outward heat transfer.

B - Temperature. Temperature changes from 20 to 40 °C. **Answer:** it changes from 293.15 to 313.15 K; the ratio is about 1.068, not 2. The difference is 20 K and 20 °C. Difference and ratio are distinct operations.

C - Cooling. In the exponential model, how long does it take to halve the difference from the environment? **Answer:** e^(−t/τ) = 1/2 gives

$t = \tau \ln(2)$, about 0.693τ. This time concerns the difference $T - T_{ext}$, not halving a whole Celsius temperature.

D - Phase change. Heat transfer is received without changing temperature. **Answer:** this does not require $Q = 0$. A phase change can absorb energy while temperature remains constant under the stated conditions. The model $mc\Delta T$ is then insufficient.

For revision, state the system, exchanges, sign convention and model's domain. Distinguish internal energy, temperature and transfer. Reconstruct a balance and then a time-dependent equation, and check units and the limit. Published official final-year framework: https://www.education.gouv.fr/bo/19/Special8/MENE1921249A.htm, appendix https://cache.media.education.gouv.fr/file/SPE8_MENJ_25_7_2019/92/9/spe249_annexe_1158929.pdf. Original LFDP writing, calculations and exercises, without reproduced text, figure or dangerous procedure.

Chapter 9 - Waves: propagation, frequency, intensity and Doppler shift

Historical reference point - 1842: frequency also depends on relative motion

The University of Vienna dates Christian Doppler's first description of the phenomenon now bearing his name to 1842. This historical reference point connects the chapter's vocabulary with a physical question: why are a periodic source and the received signal not necessarily described by the same frequency? The date of a proposal does not validate all its later uses. A current calculation still requires the medium, movement direction and sign convention to be specified; for light, an acoustic formula does not become applicable simply by renaming the wave.

Reference for the historical fact, consulted for the French manuscript on 11 October 2026: University of Vienna, history of physics, first part, https://physik.univie.ac.at/en/about-us/history/part-1/. LFDP educational analysis, without verbatim quotation or reproduced image.

1. A propagating disturbance

A wave describes the propagation of a disturbance and can transport energy. A mechanical wave requires a material medium. Local displacement of the medium's elements must not be confused with the disturbance's propagation speed. In a simple string model, a point can oscillate vertically while deformation propagates horizontally. It does not travel along the whole string with the wave.

A wave is transverse when disturbance is perpendicular to the direction of propagation, and longitudinal when these directions are parallel. Sound in a gas is described through pressure variations and local longitudinal displacements. Electromagnetic

waves do not require a material medium to propagate in a vacuum. These categories identify mechanisms; they do not say that every wave behaves identically.

The elementary model does not assume overall transport of matter identical to energy transport. More complex phenomena can involve a net displacement or flow; “no wave ever transports matter” would be too absolute. At this level, we simply distinguish local displacement, disturbance transport and possible overall movement of the medium.

2. Delay and propagation speed

If a disturbance propagates without changing shape at constant speed c, two points separated by distance d along propagation observe the same signal with delay $\tau = d/c$. Propagation speed has unit m/s; delay is a positive duration. A signal measured at the second point can be described by $s_2(t) = s_1(t - \tau)$ if time conventions and propagation direction are consistent.

For d = 1.70 m and c = 340 m/s in an acoustics exercise, delay is 0.0050 s, or 5.0 ms. This is a one-way path. For an echo received at the emission point after reflection from an obstacle at distance d, the ideal path is 2d and delay 2d/c. Confusing these geometries halves or doubles the calculated distance.

Propagation speed depends on the medium and its conditions; it is generally not the source's speed. A person walking while talking does not push all sound waves at their own speed. Source motion changes intervals between received wavefronts, leading to Doppler shift, but propagation through the medium is still described by the appropriate wave speed.

3. Period, frequency and wavelength

For a periodic wave, period T is the time separating successive identical states at a point. Frequency f = 1/T indicates cycles per second and is expressed in hertz. Wavelength λ is the distance between two points with the same phase state at a given time, for simple periodic propagation. We have λ = cT = c/f in this model.

A signal of frequency 1 000 Hz propagating at 340 m/s has wavelength 0.340 m and period 1.00 ms. Twice the frequency in the same non-dispersive medium gives half the wavelength. Period is a duration; wavelength a distance. Graphs against time and against position must therefore be read using different axes.

At a stationary interface between two media, the frequency imposed by the source generally remains unchanged in the transmission model studied, but propagation speed and wavelength can change. Higher speed in the second medium gives a longer wavelength for that frequency. Reflection and transmission can coexist; one wavelength does not describe the entire field when several waves are present.

4. Writing a sinusoidal wave and interpreting phase

A travelling wave propagating along positive x can be written s(x,t) = A cos(ωt − kx + φ), with ω = 2πf and k = 2π/λ. Amplitude A carries the unit of the disturbed quantity; phase is dimensionless. The ratio ω/k gives phase speed in this model. Changing the sign before kx reverses propagation direction while conventions are retained.

At a fixed position, a time-dependent oscillation is observed. At a fixed time, a spatial profile is observed. Two points separated by λ have the same phase; two separated by λ/2 have opposite phases for a sinusoid. This does not mean that material elements are

separated by a constant length of their own movement; it describes the signal's organisation.

If phase is held constant and time increases by Δt, position increases by (ω/k)Δt. Direction and speed are thus recovered without memorising an isolated sign. This check is useful when a different convention writes cos(kx − ωt). Because cosine is even, the forms can be equivalent with an appropriate phase; examine the whole expression.

5. Intensity and power distribution

A wave's average intensity can be defined as transported power per unit area, in W/m². For an ideal isotropic point source in a medium without absorption, power P spreads over a sphere of area 4πr². Intensity at distance r is therefore P/(4πr²). It decreases as the distribution area increases, without total power being dissipated in this model.

If distance doubles, intensity is divided by four. Starting from a fictional intensity of 10^−6 W/m² at 2 m, we obtain 2.5×10^−7 W/m² at 4 m under the same conditions. Do not apply this geometric law without examination near an extended source, in a reverberant room or in an absorbing medium. Directivity can also concentrate power in particular directions.

Absorption is another mechanism: part of the wave's energy is transferred to the medium. An attenuation law can then include an additional factor depending on distance and frequency. Distinguish geometric spreading from dissipation. Both decrease measured intensity but describe different transfers.

6. Logarithmic level and adding sources

Sound intensity level is defined by L = 10log_10(I/I_0), in decibels, with conventional reference I_0 = 10^−12 W/m² in usual exercises. The ratio is dimensionless. For I = 10^−6 W/m², the ratio is 10^6 and L = 60 dB. Level does not add like intensity, and its value alone is insufficient to describe real exposure or protection.

Doubling intensity adds 10log_10(2) ≈ 3.01 dB. Multiplying it by ten adds 10 dB. In the spherical model without absorption, doubling distance removes 10log_10(4) ≈ 6.02 dB. These differences are independent of initial level while assumptions and the same reference are retained.

For independent sources whose intensities can be added, four identical sources give four times the intensity and about 6.02 dB more than one alone. Adding their four levels would be incorrect. For coherent waves, amplitudes and phases enter interference; simple intensity addition is no longer the general description. The next chapter clarifies this condition.

The relationship between amplitude and intensity depends on the quantity and medium. For a linear wave in the same medium, intensity may be proportional to squared pressure or displacement amplitude. Doubling this amplitude then quadruples intensity rather than doubling it. Levels are no universal hearing-safety instruction; duration, spectrum and exposure conditions remain necessary.

7. Doppler shift with a moving source

Doppler shift is a difference between emitted and received frequency associated with relative motion in the geometry studied. For a sound source approaching an observer stationary in the medium, successive wavefronts are closer together. In a straight-line subsonic model, f_received = f_emitted c/(c − v_s), with v_s positive towards the observer and less than c.

The formula can be recovered. During one emitted period T_emitted, the first front advances by cT_emitted while the source advances by v_sT_emitted. The distance between two fronts ahead of it is (c − v_s)T_emitted. The stationary observer receives these fronts at speed c; frequency is c divided by this wavelength. The formula follows from geometry and period, not an arbitrary change in the source's own frequency.

With f_emitted = 440 Hz, c = 340 m/s and v_s = 20 m/s, received frequency on approach is 467.5 Hz. When receding, wavelength increases and the formula becomes 440×340/(340 + 20), about 415.6 Hz. Shifts are not exactly symmetric around 440, because source speed appears in the denominator.

8. A moving observer and electromagnetic waves

For a stationary source and observer approaching at speed v_o through the medium, the observer meets fronts at speed c + v_o. Received frequency is f_emitted(c + v_o)/c in the simple straight-line geometry. Source and observer motions therefore do not produce identical formulas for sound, because the medium provides the propagation framework.

If source and observer approach under the stated convention, combine the factors: f_received = f_emitted(c + v_o)/(c − v_s). A velocity not aligned with the line of sight requires relevant

components and precise geometry. Inserting a whole speed into a radial formula without examination can overestimate a shift.

For light in a vacuum, complete relationships require a relativistic description, different from the acoustic model with a medium. At radial speed small compared with c, an approximation gives a relative wavelength shift $\Delta\lambda/\lambda \approx v_r/c$ for recession, with v_r positive when receding. Retain the sign and approximation domain. This relationship does not cover high speeds or every spectral-shift phenomenon.

9. Superposition and standing waves

In a linear model, two disturbances can superpose by adding their values. Two waves with equal frequency and amplitude travelling in opposite directions can produce a standing wave. Some points remain nodes, with zero amplitude, while others are antinodes of maximum amplitude. A standing wave is not one travelling wave advancing as a whole in one direction.

For an ideal string of length L fixed at both ends, conditions impose nodes at the endpoints. Allowed wavelengths are $\lambda_n = 2L/n$ and frequencies $f_n = nc/(2L)$, for integers $n \geq 1$. These relationships depend on boundary conditions; a system with a free end has other modes. The word “harmonic” therefore does not remove the need to define the system.

With L = 0.50 m and c = 100 m/s in an exercise, the fundamental is 100 Hz and the third mode 300 Hz. In that mode, three half-wavelengths occupy the string. Distinguish mode number, number of nodes and number of antinodes. Endpoints are nodes; the third mode has two internal nodes and three antinodes.

10. Composite signals, beats and digitisation

A pure signal is sinusoidal in the model. A composite signal can contain several frequencies. The sum A cos(2π×400t) + 0.5A cos(2π×800t) is periodic but not a single sinusoid. The spectrum describes components and their amplitudes. Signal shape and fundamental period alone do not give every detail of the spectrum.

Two sinusoids with nearby frequencies, for example 440 and 442 Hz, can produce beats. The cosine-sum identity reveals a rapid oscillation modulated by an envelope. The frequency of observed amplitude beats is the absolute difference, here 2 Hz. This does not mean a new material source oscillating at 2 Hz was added; it arises from superposition and evolving relative phase.

To digitise a signal limited to maximum frequency f_max, sampling frequency greater than 2f_max is a fundamental condition for ideal reconstruction, with appropriate band-limited and processing assumptions. If a component exceeds this range, aliasing may make it appear at another frequency. For example, cosines at 700 and 300 Hz produce identical values at times that are multiples of 1/1 000 s. Sampled data alone cannot then distinguish them without additional information.

11. Worked exercises

A - Echo. An echo is received after 0.20 s in a model with c = 340 m/s. **Answer:** the return path is 68 m; the obstacle is at 34 m. The echo does not correspond to a one-way path of 68 m to the obstacle.

B - Wavelength. A wave has T = 0.020 s and c = 3.0 m/s. **Answer:** f = 50 Hz and λ = 0.060 m. Frequency depends on period; wavelength also uses propagation speed. Multiplying c by f would not give a length.

C - Level. Intensity is multiplied by 1 000. **Answer:** level increases by 10log_10(1 000) = 30 dB. Initial level is not multiplied by 1 000. A logarithmic scale converts a ratio into a difference.

D - Doppler. A sound source recedes from a stationary receiver. **Answer:** in the subsonic model presented, received frequency is lower. Recover the sign from more widely spaced fronts rather than memorising it without geometry.

E - Modes. An ideal string has a fundamental of 80 Hz. **Answer:** its harmonic modes are at 80, 160, 240 Hz, etc., under the same boundary conditions. A different endpoint condition requires another relationship; this result is not universal for every instrument.

12. Revision and scope

Distinguish medium motion from propagation, read a period on a time signal and wavelength on a spatial profile. Reconstruct acoustic Doppler shift using front separations. For intensity and level, always return to their definitions before adding. Also check linearity, steadiness and geometric conditions.

Published official final-year framework, waves and signals component: https://www.education.gouv.fr/bo/19/Special8/MENE1921249A.htm and appendix https://cache.media.education.gouv.fr/file/SPE8_MENJ_25_7_2019/92/9/spe249_annexe_1158929.pdf, consulted for the French manuscript on 11 October 2026. Original LFDP explanations and exercises, without reproduced third-party recording, image or question. Acoustic values are exercise parameters, not a local measurement, exposure assessment or protection instruction.

Chapter 10 - Optics: diffraction, interference, images and telescopes

Historical reference point - 1801: discussing a theory of light

The Royal Society's archives hold the text of Thomas Young's lecture on the theory of light and colours, presented on 12 November 1801; the corresponding paper was published in 1802. Distinguishing presentation from publication prevents two nearby dates from being turned into a contradiction. This milestone accompanies wave descriptions: explaining a light pattern requires comparing phases and amplitudes, not merely tracing rays. This chapter's formulas and exercises are present-day educational developments; they neither reproduce the historical apparatus nor translate its manuscript.

Reference for the historical fact, consulted for the French manuscript on 11 October 2026: Royal Society, archives L&P/11/172, https://makingscience.royalsociety.org/items/l-and-p_11_172/paper-on-the-theory-of-light-and-colours-bakerian-lecture-by-thomas-young. LFDP educational analysis, without verbatim quotation or reproduced image.

1. Several models for several questions

Light can be described by rays to study images when device dimensions are large compared with wavelength and geometric assumptions are appropriate. A wave description becomes necessary for diffraction and interference. A photon description illuminates some energy exchanges with matter. These models are not three opinions to choose between by preference; each answers questions and has a domain of validity.

A ray represents propagation direction in geometric optics, not a material thread of light. Drawing a few rays constructs an image but does not alone describe intensity distributed around it. A finite

aperture can produce a diffraction spot even when an ideal geometric diagram makes rays converge at a point. The required precision determines the necessary model.

This chapter concerns idealised devices and paper calculations. Wavelengths, sizes and focal lengths are exercise data. The formulas certify no real optical quality. Understanding the results requires no looking at the Sun, laser exposure or risky optical assembly.

2. Diffraction by an aperture

Diffraction is a change in propagation when a wave encounters an aperture or obstacle. It becomes particularly visible when the relevant dimension is comparable to wavelength. A wave therefore does not always continue as a geometric beam strictly bounded by the edges. In the wave model studied, the phenomenon does not assume random deflection of independent photons at each edge.

For a slit of width b illuminated by a monochromatic wave, the first diffraction minimum in a far-field model satisfies sin(θ) = λ/b. For small angles in radians, θ ≈ λ/b. Angle θ is the half-angle between the central axis and this first minimum. The central maximum's angular width is about 2λ/b. Confusing half-angle with total width introduces a factor of two.

On a screen at distance D under small-angle conditions, central width is $\ell \approx 2\lambda D/b$. With λ = 500 nm, b = 80 µm and D = 1.0 m, we obtain $\ell \approx 0.0125$ m, or 12.5 mm. Conversion gives 500×10^−9 m and 80×10^−6 m. A slit half as wide gives a central maximum twice as wide in this model; a longer wavelength also produces more spreading.

3. Interpreting a diffraction measurement

If the formula and conditions are appropriate, estimate an unknown width through $b \approx 2\lambda D/\ell$. Measure the central width between the first minima, rather than arbitrarily selecting two visible bright regions. Monochromatic light, geometry and distances affect the model's suitability. Light containing several wavelengths superposes several patterns and can complicate measurement.

For $\lambda = 600$ nm, $D = 2.0$ m and $\ell = 24$ mm, estimated width is 100 µm. Doubling a measurement of ℓ with other parameters unchanged would halve b. This dependence supplies a qualitative check before calculation. Negative width is impossible, and a result far from the device's dimensions calls for checking units, the definition of ℓ and assumptions.

Uncertainties in D, λ and ℓ affect b. If measured width is imprecise, adding decimal places to distance does not automatically make b precise. Present an estimate with precision compatible with the data. The model itself may be the main limiting factor if angles are no longer small or the far-field regime has not been reached.

4. Superposition and phase difference

Two superposed sinusoidal waves of the same frequency give a result depending on their phase difference. For path difference δ and sources initially in phase in the same medium, $\Delta\varphi = 2\pi\delta/\lambda$, up to a sign convention. If initial phases differ, add that difference; distance alone no longer describes interference.

Constructive interference means disturbances reinforce one another. For two equal amplitudes and $\Delta\varphi = 2m\pi$, resultant amplitude is doubled. Complete destructive interference corresponds to $\Delta\varphi = (2m + 1)\pi$ and can cancel the local

disturbance if amplitudes are equal. Path conditions are then δ = mλ and δ = (m + 1/2)λ for sources initially in phase.

For unequal amplitudes, opposite phase does not necessarily produce complete cancellation. Coherence, meaning here a phase difference sufficiently stable during observation, is also necessary for a stable pattern. Two independent sources may change relative phase too rapidly; the average interference term can then disappear. Adding intensities and adding amplitudes are not interchangeable.

5. Intensity and energy conservation

For light, the following scalar relationship assumes contributions with the same polarisation as well as coherence conditions; orthogonal polarisations do not produce this interference term. In the appropriate linear model, two waves of intensities I_1 and I_2 give locally I = I_1 + I_2 + 2sqrt(I_1I_2)cos(Δφ). If each has intensity I_0, the result becomes 4I_0cos²(Δφ/2). The local maximum is 4I_0 and the minimum zero. Two incoherent waves of equal intensity would average 2I_0 under usual addition conditions.

The higher local maximum creates no energy, and the minimum destroys none. The pattern redistributes energy in space under the arrangement's conditions. A dark and a bright region must not be studied as two independent systems each assigned all the initial energy. Conservation is checked through an appropriate balance of the whole system and its flows.

This relationship also shows why adding two decibel levels would be incorrect for interference. Work with the model's amplitudes or intensities and phases, then apply the level definition if that representation is requested. Equal frequency is insufficient: relative phase stability and geometry remain important.

6. Interference from two apertures

In an ideal arrangement of two coherent apertures separated by a, with a distant screen at D and coordinate x near the axis, path difference is approximately $\delta \approx ax/D$. For sources initially in phase, maxima are therefore separated by fringe spacing $i \approx \lambda D/a$. The approximations notably require distance large relative to a and sufficiently small angles.

With $\lambda = 600$ nm, $D = 2.0$ m and $a = 0.20$ mm, fringe spacing is 0.0060 m, or 6.0 mm. Increasing a reduces fringe spacing; moving the screen away increases it. This dependence differs from the diffraction width of an aperture of width b. Dimensions a and b have different roles and must not be exchanged in formulas.

A real double-slit pattern often has interference fringes modulated by a diffraction envelope. Fringes express path difference between apertures; the envelope depends on each finite aperture. Describing one without the other may be useful over a region, but does not reproduce the entire intensity distribution.

If i is measured to estimate λ, use $\lambda \approx ia/D$. Averaging over several fringe spacings can reduce a locating error: measuring ten spacings and dividing by ten is often more stable than measuring one. This does not remove calibration error or an inadequate geometric assumption. Improving repeatability and correcting bias are different aims.

7. Resolution limits: forming an image does not distinguish everything

A finite aperture spreads a point's image into a spot rather than a perfect mathematical point. Two objects very close in angle can produce overlapping spots. A larger aperture diameter can improve diffraction-limited resolution at a given wavelength, but aberrations, the sensor and atmosphere may impose other limits.

As an explanatory extension, a usual criterion for a circular aperture gives a characteristic angle of about 1.22λ/D_aperture in the ideal diffraction model. This number depends on the chosen criterion and aperture geometry; it promises no real telescope quality. The symbol D_aperture does not denote the screen distance in the preceding section.

Magnifying an image does not create detail that was not resolved. A greatly enlarged image can remain blurred. Distinguish magnification, resolution and brightness. This guards against simplistic interpretation of a commercially advertised magnification number, without requiring an instrument test to understand the model.

8. A thin lens and the signed convention

For a thin converging lens, choose an axis in light's propagation direction and optical centre O. Image-side focal length f' = OF' is positive. The signed lens equation is 1/OA' − 1/OA = 1/f'. For a real object on the left, OA is negative; a real image on the right has positive OA'. Signs belong to directed positions, not merely the word “distance”.

Transverse magnification is γ = A'B'/AB = OA'/OA. A negative sign indicates an inverted image; its absolute value compares sizes. A real image can be formed on a screen in the model because rays actually converge. A virtual image corresponds to intersecting extensions of rays; the same beam does not project it onto a screen placed at that position.

With f' = 0.10 m and OA = −0.30 m, the relationship gives 1/OA' = 10 − 3.333... = 6.666..., hence OA' = 0.15 m. Magnification is −0.50. The image is real, inverted and half the size. Replacing OA with positive distance 0.30 m would change the equation and give an answer incompatible with the convention.

9. Limiting cases and geometric construction

For an object at infinity in a given direction, incident rays are approximately parallel, and the image forms in the image-side focal plane. A ray parallel to the axis passes through the image-side focus after an ideal converging lens; a ray through the optical centre is undeviated in the thin-lens approximation. These rules permit construction, provided an unscaled diagram is not treated as an exact measurement.

If the object is at the object-side focus, emerging rays are parallel and the image is at infinity in the model. If the real object is between focus and lens, the image becomes virtual and upright. With f' = 0.10 m and OA = −0.050 m, we find OA' = −0.10 m and γ = +2. The image lies on the object's side and is enlarged.

The thin-lens relationship is used under paraxial conditions: rays sufficiently near the axis and modest angles, with a suitable lens model. Large angles or a wide aperture may reveal aberrations. Diffraction still limits the image even if the geometric model is solved correctly. A lens-equation calculation answers a question about ideal position and size, not every aspect of instrument quality.

10. An afocal telescope and angular magnification

An ideal astronomical telescope can be modelled by two converging lenses, an objective and an eyepiece. For an object at infinity, the objective forms an intermediate image in its image-side focal plane. In an afocal telescope, this plane coincides with the eyepiece's object-side focal plane. Rays then emerge parallel; lens separation is f'_objective + f'_eyepiece in the simple model.

Signed angular magnification is G = θ'/θ = −f'_objective/f'_eyepiece for small angles and the adopted

convention. The negative sign corresponds to an inverted image. With an objective focal length of 200 mm and eyepiece of 25 mm, ideal separation is 225 mm and G = −8. The absolute magnification is eight; it compares angles, not directly image height with the height of an object at finite distance.

A larger objective diameter can collect more light and improve a diffraction resolution limit, but does not alone change this focal-length ratio. An eyepiece changes magnification; it does not automatically repair lost information. To explain a telescope orally, distinguish the ray path, intermediate image, afocal condition and magnification.

11. Refraction, frequency and wavelength

A medium's refractive index can be defined as n = c_0/v_phase for a monochromatic light wave, with c_0 propagation speed in a vacuum. At a stationary interface, frequency stays unchanged in the transmission model, while wavelength becomes λ_medium = λ_vacuum/n. A colour related to frequency therefore does not simply become another colour because wavelength changed within the medium.

The refraction law is n_1sin(θ_1) = n_2sin(θ_2), with angles measured from the normal to the interface. Measuring from the surface without conversion is a geometry error. If n_2 > n_1, the refracted ray bends towards the normal in the usual geometry. Reflection direction and amplitude still require their own relationships.

Total internal reflection is possible when a wave passes from higher to lower refractive index and incidence exceeds a critical angle satisfying sin(θ_limit) = n_2/n_1. This condition does not apply in the opposite direction. It illustrates the importance of the media's order. Refractive index can depend on wavelength;

dispersion explains some directional differences between components of polychromatic light.

12. Worked exercises

A - Central width. λ = 400 nm, b = 0.10 mm and D = 2.0 m. **Answer:** $\ell \approx 2\lambda D/b = 0.016$ m, or 16 mm. The half-angle is about 0.004 rad. Giving width of 8 mm would omit the factor of two.

B - Fringe spacing. An ideal arrangement has i = 3.0 mm, a = 0.50 mm and D = 2.5 m. **Answer:** λ ≈ ia/D = 6.0×10^−7 m, or 600 nm. Both millimetre quantities are converted into metres before multiplication. The calculated wavelength depends on small-angle and coherence assumptions.

C - Phase. Two waves initially in phase have a path difference of 3λ/2. **Answer:** Δφ = 3π, equivalent to π modulo 2π: interference is destructive. It is completely zero only if amplitudes are equal in the model.

D - Image. f' = 0.20 m and OA = −0.40 m. **Answer:** 1/OA' = 5 − 2.5 = 2.5, hence OA' = 0.40 m and γ = −1. The image is real, inverted and the same size. Equal size does not imply identical orientation.

E - Telescope. An afocal telescope has focal lengths of 300 and 30 mm. **Answer:** ideal separation is 330 mm and signed magnification −10. Inversion follows from the model with two converging lenses; another erecting system would change the description.

F - Refractive index. A wave of given frequency has vacuum wavelength 600 nm and enters a medium of refractive index 1.5. **Answer:** its wavelength in that medium is 400 nm; frequency stays the same at the stationary interface. Phase speed is divided by 1.5.

13. Revision and references

For revision, first choose the phenomenon: aperture diffraction, interference between contributions, a lens's image formation or telescope magnification. Then write dimensions and conventions specific to it. Do not confuse slit width with slit separation, angle with screen length, or transverse with angular magnification. An annotated representation can be more instructive than an isolated formula.

Published official final-year framework: https://www.education.gouv.fr/bo/19/Special8/MENE1921249A.htm and appendix https://cache.media.education.gouv.fr/file/SPE8_MENJ_25_7_2019/92/9/spe249_annexe_1158929.pdf, consulted for the French manuscript on 11 October 2026. The circular-aperture resolution section is an identified extension; the guide does not reproduce all examination arrangements. Original LFDP explanations and exercises, without third-party textbook photographs, diagrams or quotations. Calculations constitute no assembly or optical-safety procedure.

Chapter 11 - Photons, energy levels and converting light

Historical reference point - 1905 and 1921: a law for the photoelectric effect

The Nobel Foundation's account connects Einstein's work of 1905 with explaining the photoelectric effect through light quanta. The physics prize awarded for 1921, presented the following year, notably mentions the law of this effect. It must therefore not be described as a prize for general relativity. Above all, this reference point helps us understand a change of question: increasing intensity does not necessarily replace the individual energy required to extract an electron in the single-photon model. Calculation must compare energies, not confuse beam power with frequency.

Reference for the historical fact, consulted for the French manuscript on 11 October 2026: NobelPrize.org, account of light and the Nobel archives, https://www.nobelprize.org/prizes/physics/the-dual-nature-of-light-as-reflected-in-the-nobel-archives/. LFDP educational analysis, without verbatim quotation or reproduced image.

1. Why several descriptions of light are necessary

The preceding chapter described a light wave through wavelength, phase and interference. This representation explains fringe arrangements and resolution limits. It is nevertheless insufficient to account for every way in which light and matter exchange energy. When a detector receives light, energy can be transferred through discrete events. A photon is the quantum associated with this electromagnetic interaction. Do not replace a useful theory with an opposing slogan: the phenomena studied determine the relevant quantities and model.

A photon is not a small classical ball whose trajectory could be followed without consequence in every interference device. Nor is a light wave a material substance flowing like water. Images can help an initial explanation; they cease to be useful when assigned properties foreign to the model. A good answer distinguishes what is measured, what is calculated and what belongs to an educational representation.

This guide uses photon energy to study transitions between energy levels, the photoelectric effect and some conversion balances. It does not undertake a complete construction of quantum mechanics. Numerical examples of atomic levels and materials are fictional unless otherwise stated. They allow reasoning without passing off a simplified diagram as the characteristics of a real component.

2. Energy, frequency and wavelength in a vacuum

For radiation of frequency ν, photon energy is:

$E = h\,\nu = h\,c_0 / \lambda_0$.

Planck's constant h is exactly $6.626\,070\,15 \times 10^{-34}$ J·s and light speed in a vacuum c_0 exactly $299\,792\,458$ m·s^{-1} in the International System. The notation λ_0 identifies vacuum wavelength. Frequency and wavelength are related by $c_0 = \lambda_0\,\nu$. The formula does not introduce three independent parameters: providing either spectral parameter allows the other to be calculated, then energy.

The dimension of h ν is J·s multiplied by s^{-1}, hence energy. For wavelength 600 nm, prior conversion gives $\lambda_0 = 600 \times 10^{-9}$ m. We obtain about 3.31×10^{-19} J per photon. This very small number describes an individual event; it does not mean that a source producing many photons transports negligible energy.

Passing into a transparent medium changes light's propagation speed and wavelength, but conserves frequency across a stationary interface. If refractive index is n, the medium's wavelength is $\lambda = \lambda_0/n$. Replacing λ_0 by this shortened wavelength in $h\,c_0/\lambda$ would be incorrect: it would assign the photon energy multiplied by n without a corresponding mechanism. The safest calculation uses $h\,\nu$, or $h\,c_0/\lambda_0$ when vacuum wavelength is explicitly given.

A photon has zero rest mass and non-zero energy. This does not permit applying the classical expression $m\,v^2/2$ with zero mass to conclude zero energy. That expression belongs to a different physical regime. Consistent reasoning also depends on the relationships' domains of validity.

3. The electronvolt: an energy unit, not a voltage

The electronvolt suits energies of microscopic phenomena. One electronvolt is exactly $1.602\,176\,634 \times 10^{-19}$ J. Its symbol eV must not be confused with the volt, a unit of potential difference. Their connection comes from energy associated with moving an elementary charge through a potential difference of one volt. Energy and voltage nevertheless retain different dimensions.

To convert joules into electronvolts, divide by one electronvolt's value in joules. The photon of 600 nm thus has energy of about 2.07 eV. Conversely, to convert energy of 3.0 eV into joules, multiply by 1.602×10^{-19}: the result is close to 4.81×10^{-19} J. Writing the full conversion factor prevents randomly choosing the operation's direction.

A practical shortcut, provided its units are retained, is:

E in eV ≈ 1 240 / vacuum wavelength in nm.

The number 1 240 is a rounded value of $h\,c_0$ expressed in eV·nm. It is not a new independent constant. For 400 nm, energy is about 3.10 eV; for 800 nm, about 1.55 eV. Doubling wavelength halves photon energy. This comparison provides an immediate check of every spectral calculation.

4. Beam power and photon numbers

Power measures energy transferred per unit time. In a monochromatic beam model, if each photon carries energy E and mean photon rate is $\dot{N}$, then $P = \dot{N}\,E$. We can therefore write $\dot{N} = P/(h\,\nu)$. This does not say photons arrive at regular intervals; it gives an average rate compatible with the power considered.

Take a fictional source of optical power 2.0 mW at 500 nm in a vacuum. Individual energy is close to 3.97×10^{-19} J. The average rate is therefore about 5.03×10^{15} photons per second. One millisecond represents on average some 5.03×10^{12} photons. This illustrates the contrast between microscopic energy and macroscopic flow without asking us to imagine every event.

If a source has several wavelengths, dividing total power by an arbitrarily chosen photon's energy is not exact. Distribute power according to the spectrum, calculate contributions and add them. A monochromatic approximation can sometimes be used, but it must be identified and checked against the required precision.

Increasing power at fixed frequency increases photon rate in this model; it does not change each photon's energy. Increasing frequency increases individual energy, but total power also depends on photon number. Colour and power are therefore not equivalent ways of saying “amount of light”.

5. Reading an energy-level diagram

A diagram represents a system's allowed energy states. Vertical position indicates energy, often in electronvolts. Drawn separations are quantitative only if the scale is. To study a transition, calculate the difference between final and initial energy instead of immediately seeking a spectral formula.

Consider fictional levels of −5 eV and −2 eV. Moving from the first to the second increases system energy by 3 eV. In a single-photon absorption model, the photon must supply this difference. The corresponding vacuum wavelength is about 413 nm. The reverse transition can produce a photon of the same energy if the transition considered is radiative.

Negative level energies do not make photon energy negative. They indicate the choice of energy reference, for example an unbound state's energy in a model. Transferred energy depends on the difference, and this difference is positive for the emitted photon. Shifting all level energies by the same constant preserves transitions.

Energy matching alone does not guarantee an observable transition. Selection rules, state populations, interactions and measurement conditions matter in real systems. A simple exercise uses transitions stated as possible. Interpreting a real spectrum requires references specific to the species and device studied.

A line spectrum can identify transitions; it is not always a unique label independent of every condition. Spectrometer resolution, mixtures, line broadening and the levels actually populated change its interpretation. Scientific method requires several converging indications, not one isolated visual resemblance.

6. The energy model of the photoelectric effect

The photoelectric effect is electron emission caused by radiation interacting with a material under appropriate conditions. In the elementary model where an electron receives one photon's energy, part of it crosses the barrier characterised by work function Φ. The maximum kinetic energy of emitted electrons is then:

$K_{max} = h\nu - \Phi$, when $h\nu$ is at least equal to Φ.

“Maximum” is necessary: not all electrons emerge with identical energy. Their states and losses before emission can differ. The equation describes the model's energy limit, not a law assigning the same speed to every detected electron.

For a fictional material with work function 2.2 eV and light of 400 nm, photon energy is close to 3.10 eV. The difference gives $K_{max} \approx 0.90$ eV, about 1.44×10^{-19} J. A non-relativistic approximation would give $v_{max} = \sqrt{(2 K_{max}/m_e)}$, about 5.6×10^{5} m·s^{-1}. This speed remains small compared with c_0, making the approximation consistent in this example.

Threshold frequency is $\nu_{threshold} = \Phi/h$. We can also define threshold vacuum wavelength $\lambda_{threshold} = h c_0/\Phi$. For $\Phi = 2.2$ eV, it is close to 564 nm. In the single-photon regime, a shorter wavelength supplies sufficient energy; a longer wavelength does not.

At 700 nm, individual energy is only about 1.77 eV: below the fictional work function. Simply increasing photon number does not replace each photon's missing energy in this model. Avoid making this a universal assertion about all experimental regimes: multiphoton mechanisms can occur at particular intensities, but lie outside the model studied here.

7. An experiment distinguishing frequency from intensity

To test a relationship, vary one parameter while controlling the others. At fixed frequency above the threshold, increasing photon flux can increase electrons emitted per unit time, in a regime without material or collection saturation. It does not directly change K_max in the elementary equation. At appropriate intensity, changing frequency instead changes the predicted energy maximum.

A voltage opposing electron movement can estimate their maximum energy. The positive magnitude of stopping voltage, denoted U_a, satisfies K_max = e U_a. The electron carries charge −e; potential signs in a diagram depend on their orientation. Using positive U_a avoids turning a circuit convention into negative kinetic energy.

With K_max ≈ 0.90 eV, a stopping-voltage magnitude of about 0.90 V is expected in the ideal model. This numerical correspondence uses the electronvolt's definition. It does not justify neglecting experimental effects: contact potentials, stray light, residual current and reading precision must be considered in real analysis.

Plotting K_max in joules against ν in hertz gives theoretical gradient h and intercept −Φ. If energy is in eV, gradient is expressed in eV·s, not J·s. Recognising the gradient's unit is essential when using a graph or comparing two measurements.

8. Photovoltaic conversion and correct balances

A photovoltaic cell transforms part of received light energy into usable electrical energy. Its full operation requires a model of material and charge transport more elaborate than the simple photoelectric effect. For a macroscopic balance, incident power and delivered electrical power can nevertheless be compared, provided area and conditions are specified.

A fictional area of 0.020 m² receives irradiance of 500 W·m⁻². If this is the irradiance received by the surface under the stated geometric conditions, incident power is 10 W. If useful electrical power is 1.8 W, efficiency relative to incident energy is 18 %. Another 8.2 W is distributed among reflection, possible transmission, heating and other losses according to the system.

Suppose 80 % of incident power is actually absorbed. Absorbed power is then 8 W. The fraction converted into electricity from this absorbed power is 22.5 %. The percentages do not contradict one another: their denominators differ. Efficiency presented without a system boundary or energy reference can mislead even when arithmetic is exact.

Illuminance in lux cannot be converted into power in watts through one simple constant valid for every source. Lux accounts for human visual response according to spectrum. Watt measures energy per second. Comparing sources requires knowing whether we assess visual perception, radiant power or conversion efficiency.

9. Worked questions and diagnosing errors

Exercise A. Radiation has vacuum wavelength 620 nm. Calculate photon energy in joules and electronvolts. The answer is about 3.20×10^{-19} J, or 2.00 eV. A value of 2.00 J would confuse the numerical result in eV with the result in joules; a value of 3.20×10^{-28} J would probably indicate a forgotten nanometre conversion.

Exercise B. A system model has levels −4.0 eV, −2.5 eV and −1.0 eV. What energies could correspond to differences between them? Positive differences are 1.5 eV and 3.0 eV. Two distinct transitions can therefore share the same difference of 1.5 eV. This list alone does not establish that every transition is permitted or observable.

Exercise C. A fictional material has $\Phi = 2.5$ eV. Is a photon of 500 nm sufficient in the single-photon model? Its energy is close to 2.48 eV: still slightly below threshold. Rounding too early to 2.5 eV would erase the requested distinction. Limiting wavelength is about 496 nm. The exercise recalls that an approximation suited to a general presentation can become insufficient near a threshold.

Exercise D. A monochromatic source supplies 3.0 mJ at 600 nm. Estimate the total photon number associated with this energy. Divide 3.0×10^{-3} J by 3.31×10^{-19} J, giving about 9.1×10^{15} photons. The result is a dimensionless number, not a frequency. A rate would require the emission duration.

Exercise E. At identical optical power, do sources at 800 nm and 400 nm produce the same photon rate? The photon of 400 nm carries twice the energy. Its corresponding rate is therefore half as large at equal power within the monochromatic approximation. An answer based solely on visible colour would be insufficient.

Extension - Radioactive decay, activity and half-life

Radioactivity is the spontaneous transformation of unstable nuclei, accompanied by radiation emission. A particular nucleus's decay instant cannot be predicted through a deterministic countdown. A large population nevertheless allows a statistical decay model. For one radionuclide in the simple framework studied, half-life is the duration after which the average number of remaining parent nuclei is halved. It does not mean that every nucleus disappears after two half-lives.

Consider an initial population N0, without new parent nuclei entering, and a decay constant $\lambda > 0$. The average model satisfies $dN/dt = -\lambda N$. Its solution is $N(t) = N0 \exp(-\lambda t)$, for $t \geq 0$. Here N counts only parent nuclei not yet decayed, not every nucleus in the container. Produced nuclei may themselves be radioactive: a chain's total activity then requires other equations. The simple relationship is not a universal description of all mixtures.

Activity A(t) of the parent radionuclide is the average number of decays per unit time: $A(t) = \lambda N(t)$. It is expressed in becquerels, with 1 Bq = 1 s^{-1}. To express A in Bq, λ must therefore be in s^{-1}. Activity is not a dose received by a person: radiation types, exposure and geometry require further information. No calculation in this extension determines handling instructions, a safety threshold or a clinical decision.

The relationship $N(t_{1/2}) = N0/2$ gives $\exp(-\lambda t_{1/2}) = 1/2$. Taking logarithms, $t_{1/2} = \ln(2)/\lambda$. We can then write $N(t) = N0 \times 2^{(-t/t_{1/2})}$. After two half-lives, N0/4 remains; after three, N0/8. Each interval removes half of what remains, not a constant half of N0. The curve's gradient therefore becomes less steep in absolute value, unlike linear decay.

Original fictional exercise. A parent population is modelled by $N0 = 2.0 \times 10^{12}$ and half-life of 8.0 h. Calculate λ in SI units, N after 24 h

and initial then final parent activity. Answer: 8.0 h = 28 800 s, hence $\lambda \approx 2.41 \times 10^{-5}\ s^{-1}$. After three half-lives, $N = 2.5 \times 10^{11}$. Initial activity is about 4.81×10^{7} Bq; activity after 24 h is about 6.02×10^{6} Bq, one eighth. These numbers describe only the given model; they are associated with no object to prepare or keep.

Reasoning exercise. Determine when the average population falls below 1 % of N0. The condition 2^(−t/t_1/2) < 0.01 gives t > [ln(100)/ln(2)]t_1/2, about 6.64 half-lives. For the previous data, the threshold is crossed after about 53.2 h. At seven half-lives, the fraction is 1/128, about 0.781 %. A small fraction does not mean complete disappearance and is not a safety threshold. The continuous model gives an average: for a few remaining nuclei, individual fluctuations acquire importance that this smooth curve does not represent.

Scientific references, consulted for the French manuscript on 11 October 2026: CEA, definition and radioactive decay, https://www.cea.fr/comprendre/pages/radioactivite/radioactivite.aspx?numero=1&type=chapitre ; educational document hosted by ASNR, section 1.2.7, decay and activity relationships, https://recherche-expertise.asnr.fr/sites/default/files/documents/professionnels_sante/documentation/syllabus_chapitre_1.pdf. The demonstrations and two exercises are LFDP originals. No experimental procedure, dose value or treatment information is reproduced.

10. A synthesis to explain without reciting formulas

Photon energy depends on frequency, whereas beam power also depends on photon number. An energy-level diagram allows energy differences to be calculated; whether a transition is possible does not depend on that difference alone. The elementary photoelectric effect requires comparing individual energy with work function. A photovoltaic balance compares powers defined at the same boundary and under the same conditions.

To present a solution, begin by naming the energy sought and its unit. Then specify the model: monochromatic photon, stated

transition, single-photon extraction or macroscopic conversion. Convert wavelength, retain necessary digits near a threshold, then check the result's direction. If wavelength decreases, individual energy increases. If power increases at fixed frequency, photon rate can increase. These conceptual checks prevent many calculation errors.

Framework references and constants

French specialist physics and chemistry curriculum for the final year, photon-energy and photoelectric-effect component; the discussion and exercises above are original writing, not a reproduction of the curriculum: https://www.education.gouv.fr/bo/19/Special8/MENE1921249A.htm

NIST, CODATA 2022 physical constants table, consulted for the French manuscript on 11 October 2026 for h, c_0, e and m_e: https://physics.nist.gov/cuu/Constants/Table/allascii.txt

Chapter 12 - Constructing, testing and presenting a physics solution

Historical reference point - 2018-2019: defining does not mean measuring without error

The resolution adopted by the CGPM in 2018 fixed numerical values of seven constants defining the revised SI, effective from 20 May 2019. The BIPM notably identifies Planck's constant, elementary charge and Boltzmann's constant. This institutional reference point distinguishes a value exact by definition from experimental realisation. A balance, sensor or method remains subject to uncertainty. When examining a result, therefore ask whether uncertainty concerns the defined constant, the measurement performed or your model's approximations; these three levels do not replace one another.

Reference for the historical fact, consulted for the French manuscript on 11 October 2026: BIPM, resolution 1 of the 26th CGPM, https://www.bipm.org/en/committees/cg/cgpm/26-2018/resolution-1. LFDP educational analysis, without verbatim quotation or reproduced image.

1. A physics question is not a collection of data

A problem statement can contain a description, graph, measurements, reference values and contextual details. These elements do not all have the same status. The first task is to reformulate the question as a quantity sought or a proposition to discuss. “Is the result compatible with the model?” asks for more than calculating a number. “Which device is most efficient?” requires specifying the efficiency criterion and comparison conditions.

Work therefore begins before choosing a formula. Identify the system, period studied, exchanges permitted by the model and

quantities actually available. A mass and height may suggest an energy balance, but stated friction prevents immediately concluding that mechanical energy is conserved. A resistor and capacitor suggest RC evolution, but initial state and connections must be known to determine the solution.

An effective method is to write two sentences: “I am seeking…” and “I model…”. The first avoids solving a neighbouring question; the second makes assumptions visible. These sentences can be brief in an answer. Their function is not a decorative preamble, but to establish what the solution undertakes.

The ideal models studied in this guide do not authorise independently building dangerous devices. A force, voltage, temperature or energy calculation is neither a safety procedure nor technical certification. In particular, electrical experiments, intense light sources and pressurised systems require appropriate supervision. Exercises use clearly defined theoretical situations.

2. Choosing a law by justifying its domain

Two approaches can solve the same problem with different amounts of work. To find speed after a fall without resistance, the work-kinetic energy theorem is often quicker than integrating dynamics over time. To find arrival time, energy alone is insufficient: a time relationship is needed. To study normal force during circular motion, projecting Newton's second law is more direct than a simple energy balance.

The choice therefore does not reduce to “this formula appears in this chapter”. Ask which unknowns the law connects and which it introduces. A relationship containing an unknown contact force can complicate a speed calculation when its work is zero in the model. Conversely, eliminating contact force too quickly through an

energy balance would prevent answering a question about that force.

Also distinguish a fundamental law from a constitutive relationship. Energy conservation does not depend on a perfectly linear spring. The relationship $F = -k\,x$, however, assumes a deformation regime and precise model. Similarly, charge balance remains relevant when a two-terminal component does not obey Ohm's law; its current-voltage relationship must then be replaced appropriately.

For an oral presentation, comparing two possible routes is often instructive. Explain why one directly gives the desired quantity, then use the other as an independent check where possible. Verification by a different method has more value than repeating the same calculation with numbers arranged differently.

3. A complete case: horizontal launch with air resistance neglected

A fictional object leaves a platform horizontally, 5.0 m above level ground, at speed 4.0 $m \cdot s^{-1}$. Take $g = 10\ m \cdot s^{-2}$ for simplicity and neglect air. The coordinate system has a horizontal axis pointing with motion and a vertical axis upwards; the origin is the starting point. Initial vertical velocity is zero.

The equations become $x(t) = 4\,t$ and $y(t) = -5\,t^2$. The ground corresponds to $y = -5$ m. Positive arrival time is therefore 1.0 s, then horizontal distance 4.0 m. Arrival velocity has components 4.0 and $-10\ m \cdot s^{-1}$. Its magnitude is $\sqrt{116}$, about 10.8 $m \cdot s^{-1}$, not the sum 14 $m \cdot s^{-1}$. A vector and its magnitude are not calculated in the same way.

The energy balance supplies a check: $v^2 = v_0^2 + 2\,g\,h = 16 + 100 = 116\ m^2 \cdot s^{-2}$. It recovers speed without directly determining arrival time. Agreement between the approaches confirms calculations within the model, but does not prove that air resistance is

negligible for every real object. A sheet of paper and a ball can behave differently despite the same height.

The horizontal result varies linearly with starting speed and with √h when g remains constant: x_arrival = v_0 √(2 h/g). Doubling height multiplies range by √2, not two. This dependence is more informative than a single value. It also detects an erroneous formula, such as range proportional to h when falling time should depend on √h.

4. A competing model: falling with linear drag

To illustrate another regime, now choose a downward vertical axis and resisting force −b v, with positive b. This is a linear-drag model; it does not universally describe air resistance. In some regimes, quadratic or more complex dependence would be more appropriate. Present the calculation as a model comparison, not a correction valid in every situation.

Dynamics gives m v' = m g − b v. Setting τ = m/b and v_lim = m g/b, with v(0) = 0, gives v(t) = v_lim (1 − e^(−t/τ)). Speed increases but approaches a limiting value. Acceleration is g e^(−t/τ), positive and decreasing. Initially, when t is small compared with τ, the exponential approximation gives v ≈ g t, as for falling without resistance.

Position measured from the start is z(t) = v_lim [t − τ (1 − e^(−t/τ))]. Its derivative recovers v(t), and z(0) = 0. For short times, the first-order expansion of velocity and subsequent integration give z ≈ g t^2/2. Checking initial conditions and the limiting regime is essential when an exponential expression contains several signs.

Take τ = 2.0 s and g = 10 m·s^{-2}. The fictional limiting speed is 20 m·s^{-1}. At t = 2.0 s, speed is about 12.6 m·s^{-1}, compared with 20 m·s^{-1} without resistance. Distance travelled is about 14.7 m,

compared with 20 m in the resistance-free model. Drag therefore changes both speed and the relationship between time and distance.

One measurement may be insufficient to choose between models. Seek several values over time, compare discrepancies with experimental precision and examine residual structure. A levelling curve suggests resistance, but could also result from instrument saturation. Physical interpretation must accompany an examination of the measuring device.

5. Moving from a graph to a parameter

A graph represents data with a scale, unit and sometimes a transformation. The gradient of y against x has y's unit divided by x's. Plotting velocity in m·s⁻¹ against time in seconds gives an acceleration gradient in m·s⁻². If the horizontal axis is in milliseconds, the raw gradient needs conversion before comparison with a value in SI units.

For ideal RC discharge, u(t) = u₀ e^(−t/τ). When voltages remain positive in the data used, write ln[u(t)/u₀] = −t/τ. The logarithm's quotient is dimensionless. Plotting this logarithm against time gives a line of gradient −1/τ. Using ln u without specifying a reference voltage hides this dimensional requirement, even if some numerical practices leave the reference implicit.

Suppose u(0) = 6.0 V and u(2.0 s) = 3.0 V. The model gives τ = 2/ln 2, about 2.89 s. It then predicts u(4.0 s) = 1.5 V. This third value is a testable prediction, provided it was not already used to determine τ. If measurement gives 1.9 V with small uncertainty, do not simply round more generously to save the model: examine assumptions or data.

The fit can allow a free intercept to account for an offset trigger time. Do not force the line through the origin solely because an

ideal formula suggests it, without checking that the time origin and reference voltage actually match the procedure. An assumption imposed during processing can create apparent confirmation.

6. Uncertainties, scatter and compatibility

A measured value is not exact knowledge of a quantity. Repeated measurements can describe scatter; instrument resolution, calibration, conditions and method also contribute to uncertainty. A tightly grouped series does not exclude shared bias. Ten readings with a ruler whose origin is displaced can be reproducible yet systematically too large.

Two results x_1 and x_2 can be compared with standard uncertainties u_1 and u_2. If contributions are independent, the standard uncertainty of their difference is $\sqrt{(u_1^2 + u_2^2)}$. The ratio $|x_1 - x_2|/\sqrt{(u_1^2 + u_2^2)}$ then quantifies the discrepancy relative to this scale. It helps discuss compatibility under the chosen assumptions. It does not replace bias analysis or a previously stated decision rule.

Take 2.00 s with $u_1 = 0.03$ s and 2.08 s with $u_2 = 0.04$ s. The difference is 0.08 s and its standard uncertainty 0.05 s; the ratio is 1.6. We can say the discrepancy is 1.6 times this combined uncertainty. This number alone cannot establish an exact probability that the model is true. A conclusion depends notably on what the uncertainties mean and the statistical framework adopted.

Correlation changes the calculation. If both results use the same instrument and calibration, part of their error may be shared. That contribution may cancel in a difference or add in a sum. Independent quadrature is therefore not a formula to apply automatically to every situation. In an exercise, follow the given assumptions; in a real study, ask where uncertainties originate.

7. Checking dimensions and limiting behaviour

An equation can add only quantities of the same dimension. In m g h + m v²/2, both terms are energies. In a proposed m g + m v²/2, a force would be added to an energy: the error is detectable without numerical values. Dimensional checking does not prove a formula, because different expressions may have the same dimension; it nevertheless eliminates many incorrect possibilities.

Arguments of an exponential, logarithm or trigonometric function must be dimensionless in usual models. In e^(−t/τ), the ratio of two times satisfies this condition. In cos(2π f t), frequency times time is also dimensionless. Writing e^(−t) is complete only if a time scale or unit convention has been made explicit.

Limits provide a second filter. If flow resistance increases, flow rate should not increase in a model with fixed pressure difference. If RC charging time tends to zero, voltage must return to its initial value. If observation distance increases for an ideal isotropic source, received intensity must decrease. These simple arguments rely on conditions held constant; otherwise, comparison loses meaning.

Verification must also examine extreme values that invalidate the model. In a relationship assuming small angles, choosing an angle near π/2 is not a relevant physical test of the approximation. First check whether the explored limit still belongs to the stated domain. Failure outside it informs the model's scope, not necessarily its accuracy within the domain.

8. Studying documents while keeping evidence visible

A document synthesis must distinguish what a source says from what we infer. An efficiency table measured under particular conditions does not guarantee the same efficiency in every climate. A nominal value is not a guaranteed minimum. A historical extract can explain a concept's evolution without supplying the data needed for a present-day calculation.

To compare two documents, examine dates, systems, units, balance boundaries and uncertainties. Different numbers can be compatible if they do not measure exactly the same quantity. Conversely, equal numbers can be unrelated if they concern different conditions. The first question is therefore “what is being measured?”, before “which is larger?”.

A strong argument contains a proposition, evidence and a limitation. For example: “In the isotropic model without absorption, doubling distance divides intensity by four, because it spreads over an area proportional to squared distance. This prediction does not automatically describe a reflecting environment.” A precise limitation strengthens the argument; it does not cancel it.

For an oral presentation, prepare a simple figure, a central equation and a numerical example you can reproduce. Also prepare a critical question: why neglect that interaction, choose that frame or retain that scale? An explained answer shows deeper understanding than a long calculation recited without the ability to justify it.

9. Worked synthesis exercises

Exercise A. A fictional mass of 0.40 kg descends without friction through height 1.8 m, then compresses a horizontal spring of stiffness 160 N·m⁻¹. With g = 10 m·s⁻², available energy is 7.2 J. At maximum compression, velocity is zero and k x²/2 = 7.2, hence x = 0.30 m. This assumes height no longer changes during compression and all the energy considered becomes elastic energy.

If 1.8 J is dissipated before compression, maximum elastic energy is only 5.4 J. We find x = √(10.8/160), about 0.260 m. Compression does not decrease in the same proportion as energy, because it appears squared. This explains why an overly intuitive proportional check could be wrong.

Exercise B. A sensor receives two independent sound sources, each of intensity 4.0 × 10⁻⁶ W·m⁻². With I₀ = 10⁻¹² W·m⁻², each level is about 66.0 dB. Total intensity is 8.0 × 10⁻⁶ W·m⁻² and level about 69.0 dB. Adding levels to obtain 132 dB would be false. Intensities add under this assumption; logarithmic levels are calculated afterwards.

Exercise C. A beam of 450 nm carries power of 1.0 mW. A device converts only 20 % of received photons into a useful signal, in a model where each gives one event. Individual energy is about 4.41 × 10⁻¹⁹ J, incident rate 2.27 × 10¹⁵ s⁻¹ and useful rate 4.53 × 10¹⁴ s⁻¹. Counting efficiency is not automatically the device's energy efficiency: the produced signal's energy and associated losses would need to be known.

Exercise D. Two models predict initial voltage of 8 V: the first u = 8 e^(−t/2), the second u = 8 − 2 t. At t = 0.5 s, they give about 6.23 V and 7.0 V. At t = 4 s, the second gives zero and then becomes negative, contradicting a passive discharge model whose voltage

remains positive. A linear approximation can be local; extending it indefinitely produces a false prediction.

10. A revision method that reconstructs reasoning

After an exercise, classify errors by origin: understanding the question, system and assumptions, law choice, projection or sign, algebra, conversion, calculation, interpretation. This classification helps choose a precise action. Repeating ten numerical applications does not correct confusion between acceleration and velocity; graphs and definitions must then be studied.

Revise by alternating similar exercises with new situations. The former stabilise a technique; the latter check whether you recognise when it applies. Explaining a counterexample is particularly useful: why is mechanical energy not conserved in this system, why is this flow law inappropriate, why cannot a logarithm take this quantity without a reference?

A final solution must be understandable without guessing a hidden convention. It names symbols, states assumptions, links relationships and concludes in the requested unit. It promises no more than the data allow. This requirement unifies the guide: calculation is a powerful tool when accompanied by understanding of the system, model and evidence.

Framework references

French specialist physics and chemistry curriculum for the final year, framework for experimental, modelling and communication skills; original LFDP cases and exercises: https://www.education.gouv.fr/bo/19/Special8/MENE1921249A.htm

International System of Units Brochure, BIPM, for dimensions and unit conventions: https://www.bipm.org/en/publications/si-brochure/

Appendices - Cross-topic problems and reasoned answers

A. Measuring g with a pendulum model: a methodological extension

In an extension based on the simple-pendulum model for small oscillations, the period is $T = 2\pi\sqrt{(L/g)}$. The string is assumed to have negligible mass, the moving mass is treated as a point, friction is neglected, and the amplitude is sufficiently small. This model is not a universal definition of every pendulum. An extended object or a large amplitude may require a different expression.

A fictional measurement gives $L = 0.640$ m and $T = 1.60$ s. Isolating the required quantity gives $g = 4\pi^2L/T^2$, or approximately 9.87 $m \cdot s^{-2}$. Dimensional analysis gives m/s^2, as expected. Doubling T while keeping L fixed would divide the calculated g by four; this expresses how the estimate depends on the measurement, not a way of changing gravity by changing the measurement.

For an initial uncertainty approximation with independent contributions, take $u(L) = 0.005$ m and $u(T) = 0.02$ s. The combined relative uncertainty is $\sqrt{[(u(L)/L)^2 + (2u(T)/T)^2]}$, approximately 0.0262. This gives $u(g) \approx 0.26$ $m \cdot s^{-2}$. The factor of two multiplying the relative uncertainty in T comes from the power T^{-2}. Forgetting it would underestimate this contribution.

Comparing the result with a reference value of 9.81 $m \cdot s^{-2}$ does not require declaring them equal. The difference is approximately 0.06 $m \cdot s^{-2}$, small compared with the standard uncertainty stated in this example. This compatibility does not prove that there is no bias. Checks would include the definition of L, the timing of several periods, the amplitude, and the conditions of oscillation.

Improvement means acting on an identified cause, rather than adding decimal places.

B. Determining density through apparent weighing

A fictional solid of mass 0.240 kg is fully immersed in a liquid of density 1 000 kg·m^{-3}. A device measures a vertical pulling force of 1.60 N when the solid is at rest, without touching the container. With g = 10 m·s^{-2}, its weight is 2.40 N. Equilibrium requires the pulling force plus the buoyant force to equal the weight: the buoyant force is therefore 0.80 N.

Archimedes' relation gives the immersed volume V = 0.80/(1 000 × 10) = 8.0 × 10^{-5} m³, or 80 cm³. Since the solid is fully immersed, this value is its volume within the model. Its density is 0.240/(8.0 × 10^{-5}) = 3 000 kg·m^{-3}. The result is greater than the density of the liquid, which is consistent with the need for a pulling force to hold it in place.

If the object were partly immersed, the same calculation would give only the volume of its immersed part. If a bubble adhered to the object, if the string exerted an additional force, or if the solid touched the bottom, the force balance would change. The formula used is not wrong; the quantities entering it would no longer correspond to the stated model.

This exercise distinguishes three operations: an equilibrium law gives a force, a buoyancy relation gives a volume, and a definition gives a density. Mixing these steps can conceal an assumption. Writing them separately makes it possible to check each one and explain the result orally.

C. An electrical specification is not enough to determine an energy

A fictional device supplies a constant voltage of 4.0 V at a current of 0.50 A for three minutes. The electrical power is 2.0 W and the transferred energy is 360 J, since three minutes correspond to 180 seconds. The transferred charge is 90 C. The relation energy = voltage × charge gives 360 J again under these conditions of constant voltage.

Now suppose that the voltage remains constant but the current decreases as i(t) = 0.50 e^(−t/60), with t in seconds. The energy over the same 180 seconds is the integral of 4 i(t), namely 120(1 − e^{-3}) J, approximately 114 J. Multiplying the initial power by the duration would give 360 J and greatly overestimate the transfer. Initial power is not mean power.

The mean power over this duration is approximately 0.633 W. This mean does not imply that the device actually supplied that power at every instant. It produces the same overall energy balance over the interval studied. The integral describes accumulation; the function describes local evolution.

Before using commercial battery or charger specifications, we would distinguish power, energy, nominal voltage, charge capacity, and operating conditions. The exercise establishes no operating time for a real device. It teaches us to ask for the necessary information rather than turn an isolated value into a promise of use.

D. An inverse problem may admit several explanations

Sound intensity decreases as the receiver moves further away. This observation may be compatible with geometric spreading over increasingly large surfaces, absorption by the medium, the orientation of the source, or a combination of these effects. Knowing two values is not always enough to identify the contributions separately.

In the isotropic model without absorption, $I(r) = P/(4\pi r^2)$. If the intensity at two metres is 2.0×10^{-6} W·m^{-2}, the prediction at four metres is 5.0×10^{-7} W·m^{-2}. A measurement of this value is compatible with the model, but a more complicated model could also reproduce it with certain parameters. Compatibility does not make the explanation unique.

To distinguish the models, we use several observation distances, keep the other conditions unchanged, examine directions, and assess uncertainties. We seek a prediction that differs between the hypotheses. A useful new observation is not simply a repetition at the same location; it can be chosen to distinguish the proposed mechanisms.

This reasoning also applies to a cooling curve, a trajectory, or a spectrum. A successful fit shows that the model reproduces certain data with certain parameters. A more demanding test checks data that were not used to choose those parameters. The distinction avoids calling a simple adjustment to observations already used a validation.

E. Comparing two estimates of light energy

A fictional monochromatic beam of 500 nm delivers 0.50 mJ. One method uses the known mean power over the duration; a second counts an estimated number of photons. The photon energy is approximately 3.97×10^{-19} J. The number associated with 0.50 mJ is therefore approximately 1.26×10^{15}.

If the count corrected for losses gives 1.20×10^{15} photons, its energy estimate is approximately 0.477 mJ. The values are not identical. Before concluding that they contradict each other, we would examine the uncertainties in counting and power measurement, the monochromatic nature of the source, and the definition of the duration. The calculation alone does not establish their compatibility if the uncertainties are not supplied.

A detection correction must be tied to a known efficiency under the conditions considered. Arbitrarily dividing the measured number by a percentage chosen to obtain the desired energy would be circular. We must be able to explain how the efficiency was determined and whether it depends on wavelength or flux.

The exercise connects a macroscopic energy with a microscopic description. It does not confuse photon number, power, and individual energy. The same total energy can be distributed among more lower-energy photons or fewer higher-energy photons; the spectrum determines this distribution.

F. Reading criticism and improving an answer

Initial answer: “Velocity is constant because the resultant is zero.” This sentence is correct if it refers to the velocity vector in an inertial frame and if the conditions of the model are satisfied. If the context does not specify the frame, the justification is incomplete. If it refers only to the magnitude of velocity, it can be misused for circular motion, where the resultant is non-zero despite a constant magnitude.

Improved answer: “In the chosen inertial frame, the resultant force is zero; Newton's second law therefore gives zero acceleration and a constant velocity vector.” It is slightly longer, but each clarification plays a role. Adding all the formulas in the chapter afterwards would not necessarily produce a better answer.

Initial answer: “Efficiency is 80%, so 20% of the energy disappears.” The second part is incorrect. Energy that is not useful according to the chosen criterion can be transferred in other forms or to other systems. A correct formulation specifies the input energy, the useful energy, and the boundary of the balance. The failure is not arithmetic; it comes from the interpretation of conservation.

Initial answer: “The results are close, so the model is true.” Closeness must be compared with an uncertainty and a decision criterion. Even good compatibility does not turn a model into an exhaustive description. A defensible conclusion might be: “The supplied data show no significant discrepancy according to the stated criterion and uncertainties.” This qualification is a requirement of evidence, not a lack of confidence.

G. Preparing an oral presentation with a question that opens discussion

Choose a problem limited enough to explain without hiding its assumptions. A question such as “Why does terminal speed appear in a falling model with friction?” allows you to present a system, a force, a differential equation, a solution, and a comparison with the frictionless case. It also opens a discussion about the choice of linear or quadratic friction.

Prepare a drawing in which the forces and axes are identifiable, followed by a central equation whose every term has a meaning. A numerical illustration must be described as fictional if it is. A graph must have axes and units. The presentation must not suggest that a simulation of the model is an experimental measurement.

Anticipate three questions: which phenomenon have you neglected, what observation could contradict your model, and which conclusion remains valid if a parameter changes? These questions help distinguish the structure of the reasoning from its numerical example. They allow you to demonstrate transferable understanding rather than recitation.

H. Assessing your understanding without assigning yourself an automatic score

For a chapter, try to define a concept, justify a relation, repeat a calculation, and propose a case in which the relation no longer applies. If one of these tasks fails, identify what is missing: vocabulary, representation, manipulation, or understanding of the assumptions. This diagnosis determines a targeted revision task.

A feeling of familiarity while reading is not proof of mastery. Rereading a worked answer may seem easy because the steps have already been chosen. Solving a new problem, explaining a

sign, or criticising an incorrect answer provides more demanding evidence. The effort required should not be confused with the length of the written response.

The guide offers study tools and worked answers, but reading it guarantees no examination grade. To assess your progress, explain a method without copying it, check a result by another route, and identify the limits of the model. A clear explanation, an accurate calculation, and relevant assumptions complement one another without replacing one another.